ANALYSIS AND DESIGN OF COLD ROLLING PROCESS

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By
ALOK KUMAR

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TO

MY PARENTS .

POST GRADUATE OFFICE
Submitted on 4.8.76.
21

CERTIFICATE

This is to certify that the thesis entitled "ANALYSIS AND DESIGN OF COLD ROLLING PROCESS" is a record of work carried out under my supervision and that it has not been submitted elsewhere for a degree.

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This thesis has been approved
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ALOK KUMAR

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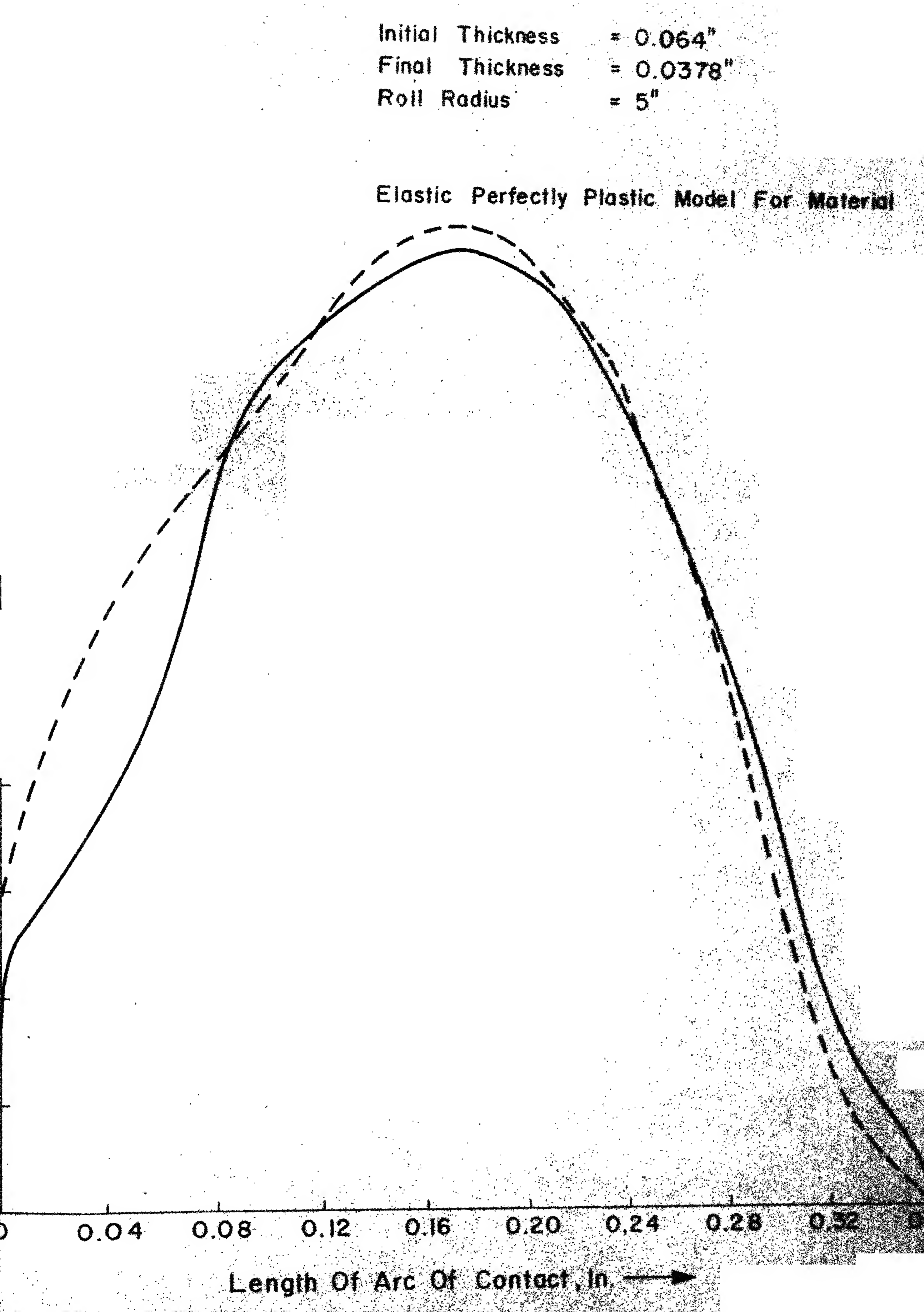

Initial Thickness = 0.064"
Final Thickness = 0.0378"
Roll Radius = 5"
Elastic Perfectly Plastic Model For Material
0.04
0.08
0.12
0.16
0.20
0.24
0.28
0.32
Length Of Arc Of Contact, In.

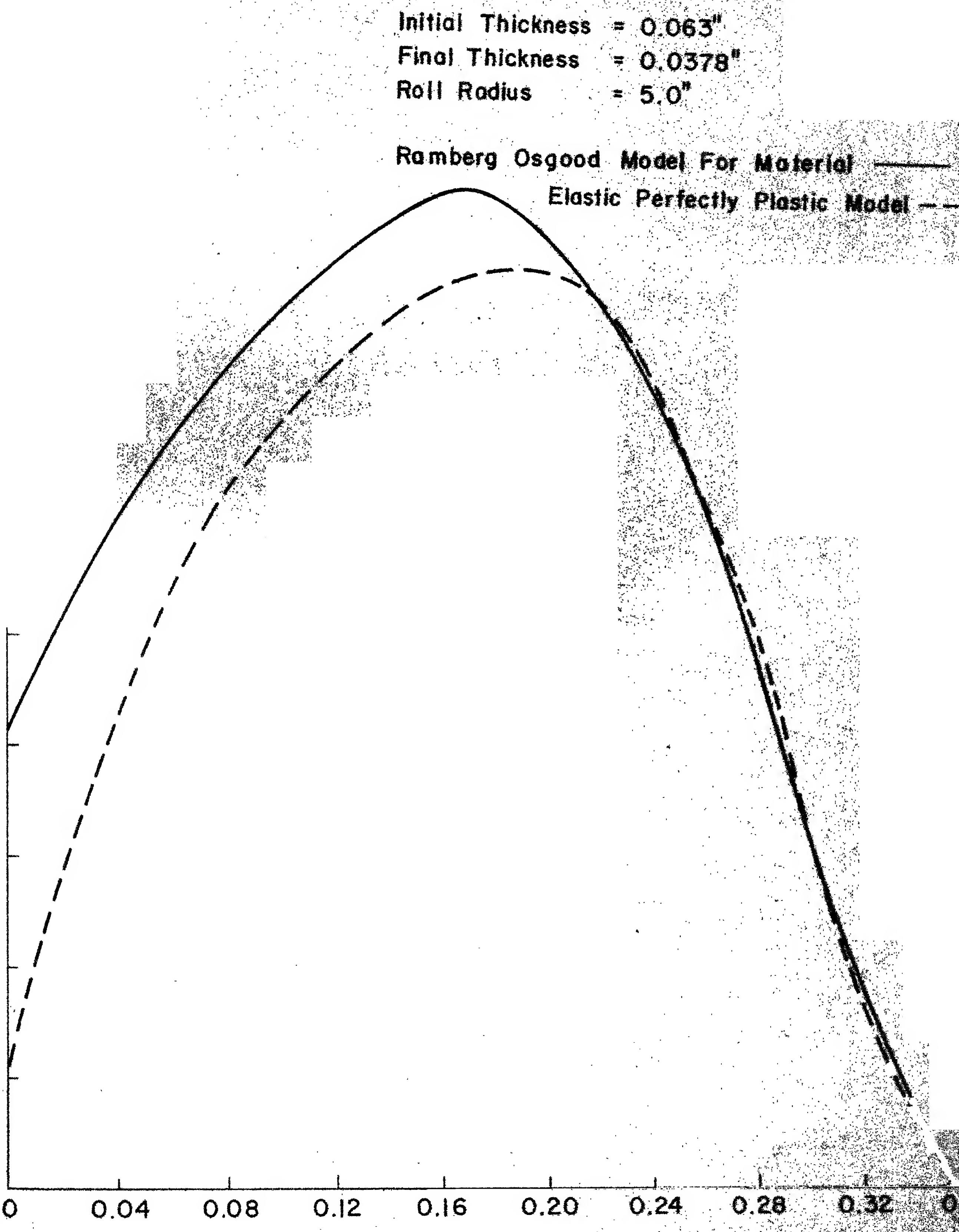

Fig.(8) Roll Pressure Diagram For Cold Rolling

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CONVERGENCE STUDY FOR FINITE ELEMENT ANALYSIS

The convergence of finite elementanalysis with respect to the number of elements is studied for Ramberg Osgood model and the results are given below :

Initial thickness of the strip = 0.063"

Final thickness of the strip = 0.0378"

S.No.	No. of Elements	No. of Degrees of freedom	Roll force (tons)
1	80	110	39.4215
2	240	310	23.9446
3	320	410	22.6285
4	400	510	18.9730

Experimental Value of Roll Force = 16.56 tons.

It can be seen that the present results with 400 elements are very close to experimental values. The discripency between experimental and theoretical values can be reduced by taking more number of elements and by taking elasticity of the rolls into account (Fig. 14).

```
14 CONTINUE
   DO19N=1,NSZF
   IF(IMP(N).EQ.0)GOTO19
   B(N)=0.0
   DO17L=1,MB
   IF(SK(N,L).EQ.0.0)GOTO17
   K=N+L-1
   B(N)=B(N)+SK(N,L)*X(K)
17 CONTINUE
   DO18L=2,MB
   K=N+1-L
   IF(K.LE.0)GOTO18
   B(N)=B(N)+SK(K,L)*X(K)
18 CONTINUE
19 CONTINUE
   PRINT22
22 FORMAT(/33H  COMPUTED COMPONENTS OF VECTOR B)
   JJ=0
   DO20I=1,NSZF
   IF(IMP(I).EQ.0)GOTO20
   JJ=JJ+1
   XX(JJ)=B(I)
    III(JJ)=I
20 CONTINUE
   PRINT16,(III(I),XX(I),I=1,JJ)
   FORMAT(*  TOTAL FORCE AT THE NODES*)
   JJ=0
   DO30I=1,NSZF
   IF(IMP(I).EQ.0)GOTO30
   JJ=JJ+1
   BB(I)=BB(I)+B(I)
   XX(JJ)=BB(I)
   III(JJ)=I
30 CONTINUE
16 FORMAT(6(I5,E15.8))
   RETURN
   END
   SUBROUTINE STRESS
   DIMENSION R(9)
   DIMENSION FINT(472,4),SINT(472,3)
   COMMON/CONTR/TITLE(12),NP,NE,NDF,NCN,NLD,NMAT,NSZF,NPRE,LMCN
   COMMON/CONTS/CORD(300,2),NCP(472,3),ORT(1,2)
   COMMON/CONTT/IMP(600),X(600),B(600)
   COMMON/CONTU/DELX,RAD,H1,H2,AD,Y
   COMMON/ABD/AA(3,6),BB(3,6)
   COMMON/ABF/FORCE(410,4),STRAN(410,3),SIGB(410),INP(410),EFST(410)
   COMMON/CDE/DIS(2,300)
   III=1
   DO90J=1,NP
   DIS(1,J)=X(III)
```

NOMENCLATURE

E = Modulus of Elasticity

ν = Poisson's ratio

σ = Stress

σ' = Deviatoric Stress

$\bar{\sigma}$ = Effective stress

ϵ = Strain

ϵ' = Deviatoric strain

$\bar{\epsilon}$ = Effective strain

$[k]$ = Total stiffness matrix

$[k^e]$ = Elastic stiffness matrix of the element

$[k^p]$ = Plastic stiffness matrix of the element

R = Undeformed roll radius

R' = Deformed roll radius

k = Yield strength at any point

h = Thickness of the strip at any point

f = Horizontal pressure in the rolled material

r = Fractional reduction in a pass

s = Normal roll pressure at any point

μ = Coefficient of friction at any point between the strip and roll

σ_1 = Back tension on the strip

σ_2 = Front tension on the strip

ϕ = Angular coordinate of a point on the arc of contact

G = Torque per unit width of material per roll

P = Roll force per unit width

P_{max} = Maximum power available to rolling mill

P_e = Roll force per unit width for elastic zone

G_e = Torque per unit width due to elastic zone

σ_b = Bending stress

τ = Torsional stress

C_{yB} = Deflection of roll due to bending

C_{yS} = Deflection of roll due to torsion

C_y = Total deflection of roll

μ' = Coefficient of friction in roll neck bearing

δ_{max} = Maximum allowable deflection in rolls

ABSTRACT

In this work the finite element method has been applied for the analysis of cold rolling process. The problem is solved by dividing the material into plane strain triangular elements. Two models of elastic-plastic behavior of the material, namely, the elastic-perfectly plastic and the Ramberg Osgood models, were considered in this work. The analysis provides the roll forces, roll torque, pressure distribution on the work-roll interface and the deformation pattern and flow of material under the rolls. The results obtained were found to be in good agreement with the available experimental results.

Further, the mathematical programming techniques were applied to design the cold rolling process optimally. The design problem is formulated using homogeneous theory of metal deformation due to Bland and Ford. The following three optimisation problems were solved:

(i) the design of single pass rolling process with the deflection of rolls as the objective for minimisation,

(ii) the design of single pass rolling process with power as the objective for minimisation,

(iii) the design of multipass rolling process with the sum of deflection of rolls in all the mills as the objective for minimisation.

It is found that tensions play a more important role in the case of rolling of thinner strips compared to that of thicker strips. Further, supplying power through the rolls has been found to be a more efficient way of transfering the power for the deformation of metals. In the case of a tandem mill, most of the reduction has been observed to take place in the first mill itself.

CHAPTER 1

INTRODUCTION

1.1 NATURE OF METAL DEFORMATION PROCESSING PROBLEM

The mechanical processes of greatest interest, from the standpoints of total tonnage handled and technological advances made, are those involving steady flow, that is, where stresses, strains and velocities at some particular reference location do not change with time. Examples include rolling, drawing and extrusion. The finite element analysis of rolling process and the design of cold rolling mills is considered in this work.

As Coffin [1] in his review puts it, "the ultimate objective in mechanical deformation processing is the attainment of a unified theory which would predict all important parameters designated by the equipment supplier, the producer and the consumer of the product." On the basis of initial properties and controllable variables the theory should provide the force, pressure distribution and power requirements necessary for the construction of equipment. Next it should describe point to point stress, strain and strain rate relationships during the deformation to

enable the producer to predict the residual stresses, the final shape of the product and whether failure would occur. Thirdly and finally, it should provide the necessary information leading to a prediction of the final properties of the product, including mechanical properties, texture and anisotropy.

The proposed mathematical model of rolling process, namely, the finite element model provides the forces, pressure distribution and torque on the basis of initial properties and variables and describes the stresses and strains at various points. Finally by knowing the strain history and stresses, metallurgists will be in a position to predict the final properties of the product.

Once the forces, pressure distribution and torques are known, it is the designer's job to design the machine with the main objective of minimising the cost (initial as well as running costs) and / or of improving the quality of the product. In metal deformation processes where large production rates are involved, the running cost assumes more importance than the initial cost. In most cases, where automation is there, the cost of running is almost directly proportional to the power requirements of the machine. For the improvement of quality, it is

desired that the machine should work near the design configuration, i.e., the deformation of components should be small. In general, the minimisation of the power and the minimisation of the deformation can not be achieved simultaneously. In fact one can be achieved only at the cost of the other.

In practice, there will be certain restrictions on the performance of the machine. These restrictions limit the choice of the design parameters. Hence the designer has to choose the design parameters so as to satisfy all the restrictions placed on the performance of the machine, which also minimise the power requirements of the machine or the deformation of the components. In the present work, the mathematical programming techniques are applied to find the design parameters which will achieve the objective stated earlier under a specified constraint set for cold rolling of strips.

1.2 LITERATURE SURVEY

1.2.1 Review of Cold Rolling Theories

The homogeneous flow theory in cold rolling assumes that the stress, strain and velocity at all points on a plane normal to the axis of the bar are constant. This assumption, along with a suitable

yield criterion, can be used to derive the differential equation for the equilibrium of a small transverse slice of the total cross section of the metal being deformed, shown in Fig. (1) (valid for wide bars, sheets and strips), as

$$d\,(fh) = s\,d\,h \pm 2\mu\,s\,d\,x = 0 \qquad (1.1)$$

This equation was first developed by Von-Karmàn [2]. Dèpending on the assumed degree of simplification of the geometry, nature of frictional effects and material properties, it is possible to solve the differential equation and obtain solution for the pressure distribution along the roll-work interface, the total roll force and the torque required. Important theories that have evolved from this approach are those of Von-Karman [2], Trinks [3], Tselikov [4], Nadai [5] and Bland and Ford [6]. In this study, Bland and Ford [6] theory is used for the optimization of the rolling process, as it is found to give good results when compared with the experimental results [21]. These theories predict the important parameters needed for the construction of equipment but, because of the assumption of homogeneous flow, are of no value in predicting the stress strain distributions during or after the defor-

The experimental investigations made by Mac Greger and Coffin [7], Hundy and Singer [8] show that non-homogeneity is present in rolling process and it is important to take it into account. The slip line-field approach is based on the very feature which produces non-homogeneous deformation and, hence, is a powerful analytical tool in metal forming processes. But according to Hill [9] accurate slip-line solution of the problem of strip rolling is not yet known and even the qualitative appearance of the slip-line field has still to be demonstrated. It is assumed to be applicable to situations where reductions are small to permit one to assume that the arc of contact is a straight line. Further, the appearance of the slip-line field is assumed to correspond to the drawing of sheets through a smooth wedge-shaped die, which gives a moderate reduction in thickness [1].

Both the homogeneous flow and the slip-line theories are applicable to plane-strain problems only. When it comes to the analysis of rolling of rectangular bars, where the spread cannot be neglected, the theories have hardly progressed beyond the empirical stage.

1.2.2 Review of Finite Element Methods in Metal Forming Processes

The finite element method was developed for the analysis of complex structures and is in the process of rapid extension to a wide range of non-structural problems. Metal forming can be considered as a non-structural problem. The finite element method has been applied to simple cases of metal forming like upsetting [10] and indentation [11], where studies were made on the development of the plastic zone, the load-displacement relationships and the stress and strain distributions. More recently finite element method is applied to the problems of plane-strain and axisymmetric extrusion [12] where the effect of frictional coefficient on the spread of plastic zone, the pressure displacement curve and the stress and strain distributions are studied for the initial non-steady state extrusion using incremental-load method of analysis.

In this study, the cold rolling of strips is analysed as a plane strain problem using the finite element method. The method is equally applicable to problems where spread can not be neglected. The incremental displacement method is used for the non-linear analysis of this problem.

1.2.3 Review of Design of Rolling Processes

The problem of design of rolling mills has always been of interest to mechanical engineers for a very long time. This is evident from the fact that many theories and empirical relations are available to calculate roll forces, pressure distribution and power requirements of the deformation process in cold rolling [2 - 6]. These quantities are very important for the design of rolling mills and / or processes. But nothing much has been done in the direction of finding optimal conditions and parameters for the rolling process. Avitzur's article [24] is one of the few attempts made in applying some theoretical considerations to the problem of tandem mill optimization in hot rolling. In this article, the rate of production is maximised. In this problem, the inter stand tensions, the thicknesses of the material in between any two adjacent mills and the speeds of the mills are taken as design variables. A method of successive approximations is used to find the optimal conditions.

In the present work, the mathematical programming techniques have been used to find the optimal design parameters for a single pass rolling mill and tandem mill for cold rolling.

1.3 DESCRIPTION OF THE PRESENT WORK

1.3.1 Finite Element Analysis

In the present work a mathematical model is proposed for the material rolled in cold rolling process using the finite element method. The material, which is to be deformed during cold rolling, is subdivided into triangular elements. Initially the material is assumed to be elastic but when it passes through the rolls it becomes plastic because of deformation. Thus the problem is treated as an elastic-plastic problem and is modelled accordingly. Depending on whether the element is in elastic or plastic state, the element stiffness matrix is calculated using the elastic or plastic stress-strain matrix and the linear strain - displacement relations. The total stiffness matrix is then assembled according to standard procedures of structural analysis. By applying the boundary conditions for forces and displacements, one is able to calculate the unknown forces and displacements. The stresses and the strains in each element can be calculated once we know the displacement vector of the continuum. This information about the stresses in each element is used for ascertaining whether the element is in elastic or plastic state and used for subsequent calculations.

As incremental displacement method is used, one can trace the stress and the strain history of each element as the material passes through the rolls. The information is useful to the producer and the consumer of the rolled material as mentioned in section 1.1.

In this analysis one can study the flow of material under the rolls and the extent of deformation of the material. Also one gets pressure diagrams for the rolls, the general shape of these diagrams is found to be in good agreement with the experimental pressure diagrams available [25] .

1.3.2 Design Optimisation

Further a design problem has been formulated as a standard optimisation problem and non-linear programming techniques have been applied to solve the problem. Two problems have been considered, one in which the material is given requisite reduction in a single pass rolling mill and the other in which the reduction is given in steps in a tandem mill. In the first problem two objectives, namely, the minimisation of power required for rolling and the minimisation of the deformation of rolls, are considered. Constraints are placed on the design parameters, which are there due to various considerations, like the stresses

developed in the rolls, location of the neutral point, the angle of bite and the deflection of rolls or the required power. An upper limit is placed on the deflection of rolls when power required for rolling is taken as the objective function and vice-versa. In the second problem, the sum of the deformations of the rolls of all the rolling mills in tandem, is taken as the objective and the problem is solved such that the solution satisfies a prescribed set of constraints. In this case, the constraints stated in the case of a single pass rolling mill need to be satisfied along with an upper bound on the reduction ratio at each mill of the tandem. Although the finite element analysis is not used to calculate roll forces, torques etc. in the present design problem, it can be done* without much difficulty to predict the behaviour of the rolls more accurately.

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*It requires more computer time and storage

CHAPTER 2

FINITE ELEMENT ANALYSIS OF COLD ROLLING PROCESS

2.1 INTRODUCTION

The first step in the finite element analysis involves the discretization of the continuum. In this study, the problem of cold rolling is treated as a plane-strain problem. This is justified for strip or sheet rolling which involves negligible spread compared to the deformation in the other two directions. For a two dimensional problem, one has a choice between triangular and quadrilateral elements for dividing the continuum.

The triangular element is the most obvious choice in most of the studies as it is simple and versatile for representing arbitrary geometries. Further even if one approximates the region by quadrilateral or higher order polygons, they can always be subdivided into a finite number of triangles. Besides, the approach using triangular, rather than quadrilateral, elements is much simpler to implement computationally [15]. In a triangular element, the displacement function can be assumed to be linear as:

$$f(x, y) = a + bx + cy \qquad (2.1)$$

whereas for a rectangular element, the displacement function has to be bilinear as

$$f(x, y) = a + bx + cy + dxy \qquad (2.2)$$

Thus all the subsequent analysis and computation is altered because of xy - term. Hence a plane constant-strain triangular element is chosen to discretize the continuum.

2.2 VIRTUAL WORK FORMULATION

The 'prescriptions' given by Zienkiewicz [16] for deriving the characteristics of a finite element of a continuum is given here in mathematical form.

A typical finite element is defined by nodes i, j, m .... and straight line boundaries. Let the displacement of any point within the element be defined as column vector $\{f(x, y)\}$

$$\{f\} = [N]\{\delta\}^e = [N_i \; N_j \; N_m \; \ldots] \begin{Bmatrix} \delta_i \\ \delta_j \\ \delta_m \\ \vdots \end{Bmatrix}^e \qquad (2.3)$$

in which the components of $[N]$ are in general function of position and $\{\delta\}^e$ represents a listing of nodal displacements for a particular element.

With the displacements known at all points within the element, the strains at any point can be determined. These will always result in a relationship which can be written in matrix notation as :

$$\{\epsilon\} = [B]\{\delta\}^{e} \qquad (2.4)$$

Assuming an elastic behavior, the relationship between stresses and strains will be linear and of the form

$$\{\sigma\} = [D]^{e} \left(\{\epsilon\} - \{\epsilon_o\} \right) \qquad (2.5)$$

Where $[D]^{e}$ is an elasticity matrix containing the appropriate material properties, $\{\epsilon\}$ is the vector of actual strains and $\{\epsilon_o\}$ is the vector of initial strains.

Let $\{F\}^{e}$ define the nodal forces which are equivalent statically to the boundary stresses and distributed loads on the element :

$$\{F\}^{e} = \begin{Bmatrix} F_i \\ F_j \\ F_m \\ \cdot \\ \cdot \\ \cdot \end{Bmatrix} \qquad (2.6)$$

Each of the forces $\{F_i\}$ must contain the same number of components as the corresponding nodal displacements $\{\delta_i\}$ and be ordered in the appropriate, corresponding directions.

The distributed loads $\{p\}$ are defined as those acting on a unit volume of material within the element with directions corresponding to those of the displacement $\{f\}$ at that point.

To make the nodal forces statically equivalent to the actual boundary stresses and distributed loads, the principle of virtual work is used. According to this principle, the external and the internal works done by the various forces and stresses are equated during an arbitrarily imposed (virtual) displacements of the nodes.

Let such a virtual displacements be $\{\delta^*\}^e$ at the nodes. This results by Eq. (2.3) and Eq. (2.4) in forces and strains within the element equal to

$$\{f^*\} = [N]\{\delta^*\}^e \quad \text{and} \quad \{\epsilon^*\} = [B]\{\delta^*\}^e \tag{2.7}$$

respectively.

The work done by the nodal forces is given by

$$(\{\delta^*\}^e)^T \cdot \{F\}^e \tag{2.8}$$

Similarly the internal work per unit volume done by the stresses and the distributed forces is

$$\{\epsilon^*\}^T \{\sigma\} - \{f^*\}^T \{p\} \tag{2.9}$$

or $$(\{\delta^*\}^e)^T ([B]^T . \{\sigma\} - [N]^T . \{p\}) \tag{2.10}$$

By equating the external work to the total internal work, obtained by integrating over the volume of the element, one obtains

$$(\{\delta^*\}^e)^T \{F\}^e = (\{\delta^*\}^e)^T (\int [B]^T \{\sigma\} d\,(vol) - \int [N]^T \{p\}\, d\,(vol)) \tag{2.11}$$

This relation is valid for any value of the virtual displacement and hence the equality of the multipliers must exist. On substitution from Eq. (2.4) and Eq. (2.5), this leads to

$$\{F\}^e = (\int [B]^T [D]^e [B] \; d\,(vol)) \{\delta\}^e - \int [B]^T [D]^e \{\epsilon_o\} \; d\,(vol) - \int [N]^T \{p\}\, d\,(vol) \tag{2.12}$$

This elemental relationship can be expressed as:

$$\{F\} = [K] \{\delta\} + \{F\}_p + \{F\}_{\epsilon_o} \tag{2.13}$$

where $\{F\}_p$ represents the nodal force required to balance any distributed loads acting on the element and $\{F\}_{\epsilon_o}$ the nodal force required to balance any

initial strains, such as may be caused by temperature changes, if the nodes are not subjected to any displacements. The first term in the right hand side of Eq. (2.13) represents the force induced by the displacement of the nodes. It can be seen that the stiffness matrix is given by

$$[K]^{e} = \int [B]^{T} [D^{e}][B] \; d \; (vol), \tag{2.14}$$

the nodal forces due to the distributed loads by

$$\{F\}^{e}_{p} = -\int [N]^{T} \{p\} \; d \; (vol), \tag{2.15}$$

and the nodal forces due to the initial strains by

$$\{F\}^{e}_{\epsilon_o} = -\int [B]^{T} [D^{e}] \{\epsilon_o\} \; d \; (vol) \tag{2.16}$$

The same results can also be arrived at by use of the principle of minimum potential energy.

2.3 ELASTIC PLASTIC ANALYSIS AS NON-LINEAR PROBLEM

In finite element analysis, non-linearities occur in two different forms. The first is due to material or physical non-linearity and the second is due to geometric non-linearity.

The first category encompasses the problems in which the stresses are not linearly proportional to

the strains, but in which only small displacements and small strains are considered. Many significant engineering problems fall under this category and the elastic plastic analysis is one of them. In elastic-plastic analysis, the elastic stiffness matrix is usually replaced by plastic stiffness matrix when the element is transformed from elastic to plastic state.

2.3.1 Derivation of Elastic Stiffness Matrix

In this section the stiffness matrix for plane constant-strain triangular element in elastic state is derived.

Fig. (2) shows a typical triangular element with nodes i, j, m numbered in an anti-clockwise order.

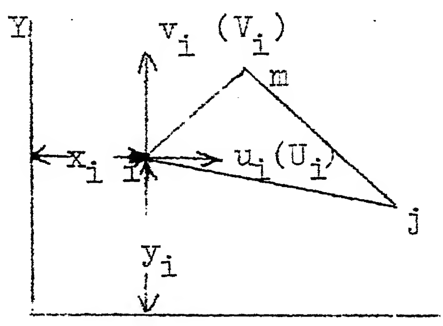


Figure (2)

The displacement vector of a node has two components as:

$$\left\{\delta_i\right\} = \left\{\begin{matrix} u_i \\ v_i \end{matrix}\right\} \tag{2.17}$$

and the six components of displacements of the element are listed as:

$$\left\{ \delta \right\}^e = \left\{ \begin{array}{c} \delta_i \\ \delta_j \\ \delta_m \end{array} \right\} \qquad (2.18)$$

The displacement within an element has to be uniquely defined by these six values. The simplest representation is clearly given by two linear polynomials as:

$$\begin{aligned} u &= \alpha_1 + \alpha_2 x + \alpha_3 Y \\ v &= \alpha_4 + \alpha_5 x + \alpha_6 Y \end{aligned} \qquad (2.19)$$

The six constants α_i, $i = 1, 6$, can be evaluated easily by solving the two sets of three simultaneous equations, which will arise if the nodal co-ordinates are inserted and the displacements equated to the appropriate nodal displacements. One finally gets

$$\begin{aligned} u &= \frac{1}{2\Delta} \left(a_i + b_i x + C_i y \right) u_i \\ &\quad + (a_j + b_j x + C_j y) u_j + (a_m + b_m x + C_m y) u_m \\ v &= \frac{1}{2\Delta} (a_i + b_i x + C_i y) v_i \\ &\quad + (a_j + b_j x + C_j y) v_j + (a_m + b_m X + C_m y) u_m \end{aligned}$$

$$(2.20)$$

where

$$a_i = x_j y_m - x_m y_j$$
$$b_i = y_j - y_m \qquad (2.21)$$
$$C_i = x_m - x_j$$

With the other coefficients obtained by a cyclic permutation of subscripts in the order i, j, m and

$$2\Delta = \det \begin{vmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_m & y_m \end{vmatrix} = 2 \text{ (Area of triangle ijm)} \qquad (2.22)$$

The total strain at any point with in the element can be defined by its three components which contribute to the internal work as :

$$\{\epsilon\} = \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{Bmatrix} \qquad (2.23)$$

Using Eq. (2.20) one gets

$$\{\epsilon\} = \frac{1}{2\Delta} \begin{bmatrix} b_i & 0 & b_j & 0 & b_m & 0 \\ 0 & C_i & 0 & C_j & 0 & C_m \\ C_i & b_i & C_j & b_j & C_m & b_m \end{bmatrix} \{\delta\}^e \qquad (2.24)$$

which defines the matrix $[B]$ of Eq.(2.4) explicitly.

In plain strain, a normal stress σ_z exists in addition to the three other stress components. For isotropic materials, one gets

$$\left.\begin{aligned}
\epsilon_x &= \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right] \\
\epsilon_y &= \frac{1}{E}\left[\sigma_y - \nu(\sigma_z + \sigma_x)\right] \\
\epsilon_{xy} &= \frac{2(1+\nu)}{E}\tau_{xy} \\
\epsilon_z &= \frac{1}{E}\left[\sigma_z - \nu(\sigma_x - \sigma_y)\right] = 0
\end{aligned}\right\} \qquad (2.25)$$

On eliminating σ_z and solving for three remaining stresses, one gets the matrix $[D^e]$ as :

$$[D^e] = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \qquad (2.26)$$

The stiffness matrix of the element ijm is defined from the general relationship of Eq. (2.14) as:

$$[K]^e = \int [B]^T [D^e] [B] \; t \, dx \, dy \qquad (2.27)$$

where t is the thickness of the element and the integration is taken over the area of the triangle. If thickness t is constant over the element, and, as neither of the matrices contains x or y, one gets

$$[K]^e = [B]^T [D^e] [B] \; t \qquad (2.28)$$

This form is now sufficiently explicit for computation.

2.3.2 Derivation of Plastic Stiffness Matrix

In applying the incremental procedure, the elastic plastic constitutive law can be expressed in terms of finite increments as :

$$\{\Delta\sigma\} = [D^p]\{\Delta\epsilon\} \qquad (2.29)$$

The matrix $[D^p]$ is updated for each increment by modifying its components because of the change in the value of the stresses. Once $[D^p]$ is obtained, the element stiffness can be computed by writing

$$[K]^e = \int [B]^T [D^p][B] \, d\,(\mathrm{vol}) \qquad (2.30)$$

For a triangular element it becomes

$$[K]^e = [B]^T [D^p][B] \, . \, t \, . \, \Delta \qquad (2.31)$$

The present formulation relies on the plastic stress-strain matrix $[D^p]$ for the yielded elements, where $[D^p]$ takes the roles of $[D^e]$. An explicit expression for the matrix $[D^p]$ for Von-Mises material is presented in this section [14].

The Prandtl-Reuss equation for the deviatoric strain increment $d\epsilon'_{ij}$ during continued loading is [9]

$$d\epsilon'_{ij} = \sigma'_{ij}\, d\lambda + \frac{d\,\sigma'_{ij}}{2G} \qquad (2.32)$$

where

$$d\lambda = \frac{3}{2} \cdot \frac{d\bar{\epsilon}^{p}}{\bar{\sigma}} = \frac{3}{2} \cdot \frac{d\dot{\bar{\sigma}}}{\bar{\sigma} H'} \tag{2.33}$$

Adopting the usual summation convention the equivalent stress $\bar{\sigma}$ and the plastic strain increment $d\bar{\epsilon}^{p}$ are expressed as follows

$$\bar{\sigma} = \sqrt{\left(\frac{3}{2} \sigma'_{ij} \cdot \sigma'_{ij} \right)}, \quad d\bar{\epsilon}^{p} = \sqrt{\left(\frac{2}{3} d\epsilon^{p}_{ij} \cdot d\epsilon^{p}_{ij} \right)} \tag{2.34}$$

In Eq. (2.33) $H' = \frac{d\bar{\sigma}}{d\bar{\epsilon}^{p}}$ corresponds to the slope of the equivalent stresss ($\bar{\sigma}$) - plastic strain ($\int d\bar{\epsilon}^{p}$) curve.

The Von-Mises yield criterion and its differential form are respectively given by

$$\sigma'_{ij} \; \sigma'_{ij} = \frac{2}{3} \bar{\sigma}^{2} \tag{2.35}$$

and

$$\sigma'_{ij} \, d\sigma'_{ij} = \frac{2}{3} \bar{\sigma} \, d\bar{\sigma} = \frac{4}{9} \bar{\sigma}^{2} H' d\lambda \tag{2.36}$$

Eliminating $d\sigma'_{ij}$ from Eq. (2.32) and Eq. (2.36),

$$2G \, \sigma'_{ij} (d\epsilon'_{ij} - \sigma'_{ij} d\lambda) = \frac{4}{9} \bar{\sigma}^{2} H' d\lambda$$

Solving for $d\lambda$ and making use of the relation (2.35), one gets

$$d\lambda = \frac{\sigma'_{ij} \, d\epsilon'_{ij}}{S} = \frac{\sigma'_{ij} \, d\epsilon_{ij}}{S} \tag{2.37}$$

with

$$S = \frac{2}{3} \bar{\sigma}^2 \left(1 + \frac{H'}{3G}\right) \qquad (2.38)$$

Note the equality $\sigma'_{ij} \, d\epsilon'_{ij} = \sigma'_{ij} \, d\epsilon_{ij}$ in the Eq. (2.37), since $\sigma'_{ij} = \sigma'_x + \sigma'_y + \sigma'_z$ is identically zero.

Now substituting $d\lambda$ of Eq. (2.37) back into Eq. (2.32) and recalling the definition for the deviatoric strain-increment $d\epsilon'_{ij}$:

$$d\epsilon'_{ij} = d\epsilon_{ij} - \delta_{ij} \frac{d\epsilon_{ii}}{3} , \quad d\epsilon_{ii} = d\epsilon_x + d\epsilon_y + d\epsilon_z ,$$

the deviatoric stress increment $d\sigma'_{ij}$ can be expressed as

$$d\sigma'_{ij} = 2G \left(d\epsilon'_{ij} - \sigma'_{ij} \frac{\sigma'_{kl} \, d\epsilon_{kl}}{S}\right)$$

$$= 2G \left(d\epsilon_{ij} - \delta_{ij} \frac{d\epsilon_{ii}}{3} - \sigma'_{ij} \frac{\sigma'_{kl} \, d\epsilon_{kl}}{S}\right) \qquad (2.39)$$

The identity $\sigma'_{ij} \, d\epsilon_{ij} = \sigma'_{kl} \, d\epsilon_{kl}$ has been used in the above expression.

The total stress increment $d\sigma_{ij}$ is, by definition,

$$d\sigma_{ij} = d\sigma'_{ij} + \frac{E}{3(1-2\nu)} \delta_{ij} \, d\epsilon_{ii}$$

$$= d\sigma'_{ij} + \frac{2(1+\nu) \cdot G}{3(1-2\nu)} \delta_{ii} \, d\epsilon_{ii}$$

Finally substituting for $d\,\sigma'_{ij}$ from Eq. (2.39), one gets

$$d\,\sigma_{ij} = 2G\left(d\epsilon_{ij} + \frac{\nu}{1-2\nu}\,\delta_{ii}\,d\epsilon_{ii} - \sigma'_{ij}\,\frac{\sigma_{kl}\,d\epsilon_{kl}}{S}\right) \qquad (2.40)$$

Eq. (2.40) is represented in matrix form as

$$\{d\,\sigma\} = [D^{p}]\,\{d\,\epsilon\} \qquad (2.41)$$

The plastic stress-strain matrix is symmetric and expressed in the explicit form as :

$$[D^{p}] = \frac{E}{(1+\nu)}\begin{bmatrix} \frac{1-\nu}{1-2\nu}-\frac{\sigma_x'^{2}}{S} & & & & & \text{Symmetric} \\ \frac{\nu}{1-2\nu}-\frac{\sigma_x'\sigma_y'}{S} & \frac{1-\nu}{1-2\nu}-\frac{\sigma_y'^{2}}{S} & & & & \\ \frac{\nu}{1-2\nu}-\frac{\sigma_x'\sigma_z'}{S} & \frac{\nu}{1-2\nu}-\frac{\sigma_y'\sigma_z'}{S} & \frac{1-\nu}{1-2\nu}-\frac{\sigma_z'^{2}}{S} & & & \\ -\frac{\sigma_x'\tau_{xy}}{S} & -\frac{\sigma_y'\tau_{xy}}{S} & -\frac{\sigma_z'\tau_{xy}}{S} & \frac{1}{2}-\frac{\tau_{xy}^{2}}{S} & & \\ -\frac{\sigma_x'\tau_{yz}}{S} & -\frac{\sigma_y'\tau_{yz}}{S} & -\frac{\sigma_z'\tau_{yz}}{S} & -\frac{\tau_{xy}\tau_{yz}}{S} & \frac{1}{2}-\frac{\tau_{yz}^{2}}{S} & \\ -\frac{\sigma_x'\tau_{zx}}{S} & -\frac{\sigma_y'\tau_{zx}}{S} & -\frac{\sigma_z'\tau_{zx}}{S} & -\frac{\tau_{xy}\tau_{zx}}{S} & -\frac{\tau_{yz}\tau_{zx}}{S} & \frac{1}{2}-\frac{\tau_{zx}^{2}}{S} \end{bmatrix}$$

For a plane-strain problem, it reduces to

$$[D^p] = \frac{E}{1+\nu}\begin{bmatrix} \frac{1-\nu}{1-2\nu} - \frac{\sigma_x'^2}{S} & & \text{Symmetric} \\ \frac{\nu}{1-2\nu} - \frac{\sigma_x'\sigma_y'}{S} & \frac{1-\nu}{1-2\nu} - \frac{\sigma_y'^2}{S} & \\ \frac{-\sigma_x'\tau_{xy}}{S} & \frac{-\sigma_y'\tau_{xy}}{S} & \frac{1}{2} - \frac{\tau_{xy}^2}{S} \end{bmatrix} \tag{2.43}$$

and $d\ \sigma_z$ is given by

$$d\ \sigma_z = \frac{E}{(1+\nu)}\ \left(\frac{\nu}{1-2\nu} - \frac{\sigma_x'\sigma_z'}{S}\right) d\,\epsilon_x + \left(\frac{\nu}{1-2\nu} - \frac{\sigma_y'\sigma_z'}{S}\right) d\,\epsilon_y + \left(-\frac{\sigma_z'\tau_{xy}}{S}\right) d\,\epsilon_{xy} \tag{2.44}$$

Eq. (2.41) corresponds to the inverse of the complete stress-strain relation [9] of Prandtl-Reuss. The original stress-strain relations have been reduced to the single Eq. of (2.41) and it must be emphasize that the elastic compressibility as well as strain-hardening characteristics of the material are incorporated in the matrix $[D^p]$. By comparing Eq. (2.43) with Eq. (2.26), it can be seen that the diagonal elements of $[D^p]$ are definitely less than the corresponding diagonal elements of $[D^e]$. It amounts to an apparent decrease of stiffness or rigidity due to plastic yielding.

2.3.3 Criterion for Yielding-

A law defining the limit of elasticity under any possible combination of stresses is known as the criterion for yielding or yield criterion. The two simplest and most widely used criteria, which are consistent with experimental evidence, are the criterion of Tresca and criterion due to Von-Mises. For most metals, Von-Mises law fits the experimental results more closely than Tresca's, but Tresca's criterion is simpler to use in theoretical applications. In this work, the Von-Mises yield criterion has been utilized. For a general state of stress, in terms of the cartesian coordinates, this criterion postulates that yielding occurs when

$$\bar{\sigma}^2 = \frac{1}{2}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2\right] + 3.0\left[\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2\right] = 3\,k^2 \qquad (2.45)$$

where, k is a parameter dependent on the amount of prestrain. k is taken $(Y/\sqrt{3})$ when Y is the yield stress of the material in uniaxial tension.

2.3.4 Elastic Plastic Behavior of Materials

The problem is solved for two non-linear models of the material. Firstly the problem is solved for the elast-perfectly plastic behavior of the material for which

$$H' = \frac{d\bar{\sigma}}{d\bar{\epsilon}^p} = 0 \tag{2.46}$$

The second model considered in this work is of the type discussed by Ramberg and Osgood [19] for explaining the tensile stress-strain relations for certain materials. Ramberg and Osgood have shown that most of the stress-strain curves in tension can be represented by the relation

$$\epsilon = \frac{\sigma}{E}\left(1 + \alpha\left(\frac{\sigma}{Y}\right)^{n-1}\right) \tag{2.47}$$

where $\alpha \simeq 0.02$ corresponds to the usual engineering definition of yielding. Tensile curves for $\alpha = 0.02$ are shown in Fig. (4b) for different values of n.

H' can be derived from Eq. (2.47) as:

$$H' = \frac{E}{n\alpha}\left(\frac{Y}{\sigma}\right)^{n-1} \tag{2.48}$$

2.4 COMPUTATIONAL PROCEDURE

We start with the strip of material divided into triangular finite elements as shown in Fig. (3). The strip is rolled through the rolls as it passes. The boundary conditions are taken as follows:

$F_x = 0$, $F_y = 0$, external forces on all the nodes except those which are under the rolls and those lying on the line CD,

$F_x = 0$, $v = 0$ on the nodes lying on the line CD, the displacement of the nodes which are in contact of the rolls are

$u = 0$

$v =$ vertical displacement of nodes because of the displacement ΔX in X direction

$$= \Delta y = \frac{h_2}{2} + R - \sqrt{R^2 - B'E'^2} - y_i \tag{2.49}$$

where

$$B'E' = CD + \sqrt{R(h_1 - h_2) - (h_1 - h_2)^2 / 4} - (x_i + \Delta x)$$

(x_i, y_i) are coordinates of the node i in contact with the rolls and CD as shown in Fig. (3) is the length of the strip considered for the analysis. See Appendix (A) for the derivation of v.

The computation starts from the stress-free condition and the displacement is given to the material

in X direction in an incremental way. The following assumptions are made in this procedure:

(i) The arc of contact does not change, i.e., the rolls are rigid.

(ii) The spread is zero, hence the problem can be treated as plane-strain problem.

(iii) To plot the pressure diagram, the radial pressure at any point along the arc of contact is equal to its vertical component.

Taking the above assumptions, the initial non-steady state of cold rolling is analyzed.

One thing needs to be noted in this problem is that for each node, either the force or the displacement component is known and hence one solves the simultaneous equations of the system for the unknown force and displacement components.

A flow chart for the computer program developed for this analysis is given in Fig. (5). The total stiffness matrix [k] is stored in the banded form and the nodes are numbered in such a way as to obtain the least band width 'B'. A small band width is usually obtained simply by numbering nodes along the shortest dimension of the continuum. The banded form storage of the [k] matrix requires only NB

locations as compared to about $N^2/2$ locations for storage of the upper or lower triangle of a full symmetric matrix of order 'N'.

Also of importance is the savings in execution time that band operations permit. A band-form Gauss elimination solver, discussed below, is used to solve the simultaneous equations that result after the incorporation of the boundary conditions. By counting the basic operations, i.e., one multiplication or division plus one addition or subtraction, the solution time can be estimated to be approximately proportional to $NB^2/2$ in the case of band matrix as compared to a value of $N^3/6$ for a full symmetric matrix.

The Gauss elimination method is applied to solve the simultaneous equations.

$$\left[\begin{array}{c:c} k_{11} \quad n \times n & k_{12} \quad N-n \times n \\ \hdashline k_{21} \quad n \times N-n & k_{22} \quad N-n \times N-n \end{array}\right] \left\{\begin{array}{c} \vec{u}_1 \\ \hdashline \vec{u}_2 \end{array}\right\} = \left\{\begin{array}{c} \vec{F}_1 \\ \hdashline \vec{F}_2 \end{array}\right\} \qquad (2.50)$$

where $\vec{u}_1$ and $\vec{F}_2$ vectors of displacement and force components are known respectively. Firstly we substitute the value of known $\vec{u}_1$ in the equations corresponding to known $\vec{F}_2$. Thus we get (N - n)

equations in (N - n) unknowns $\vec{u}_2$. These simultaneous equations are solved by Gauss band form elimination method, the details of which are given in Cook [17]. Once we know $\vec{u}_2$, the unknown vector of modal forces $\vec{F}_1$ is calculated from the displacement vector $\vec{u} = \begin{Bmatrix} \vec{u}_1 \\ \vec{u}_2 \end{Bmatrix}$ by substituting the values in first n simultaneous equations of Eq. (2.50).

Once the nodal displacement vector is known, one gets the new coordinates of nodal points. The strains and the stresses in the elements can be obtained from

$$\{\epsilon\} = [B] \{\vec{u}\} \qquad (2.51)$$

and $$\{\sigma\} = [D] [B] \{\vec{u}\} \qquad (2.52)$$

The listing of the computer program developed for the analysis of cold rolling process is given in Appendix (D).

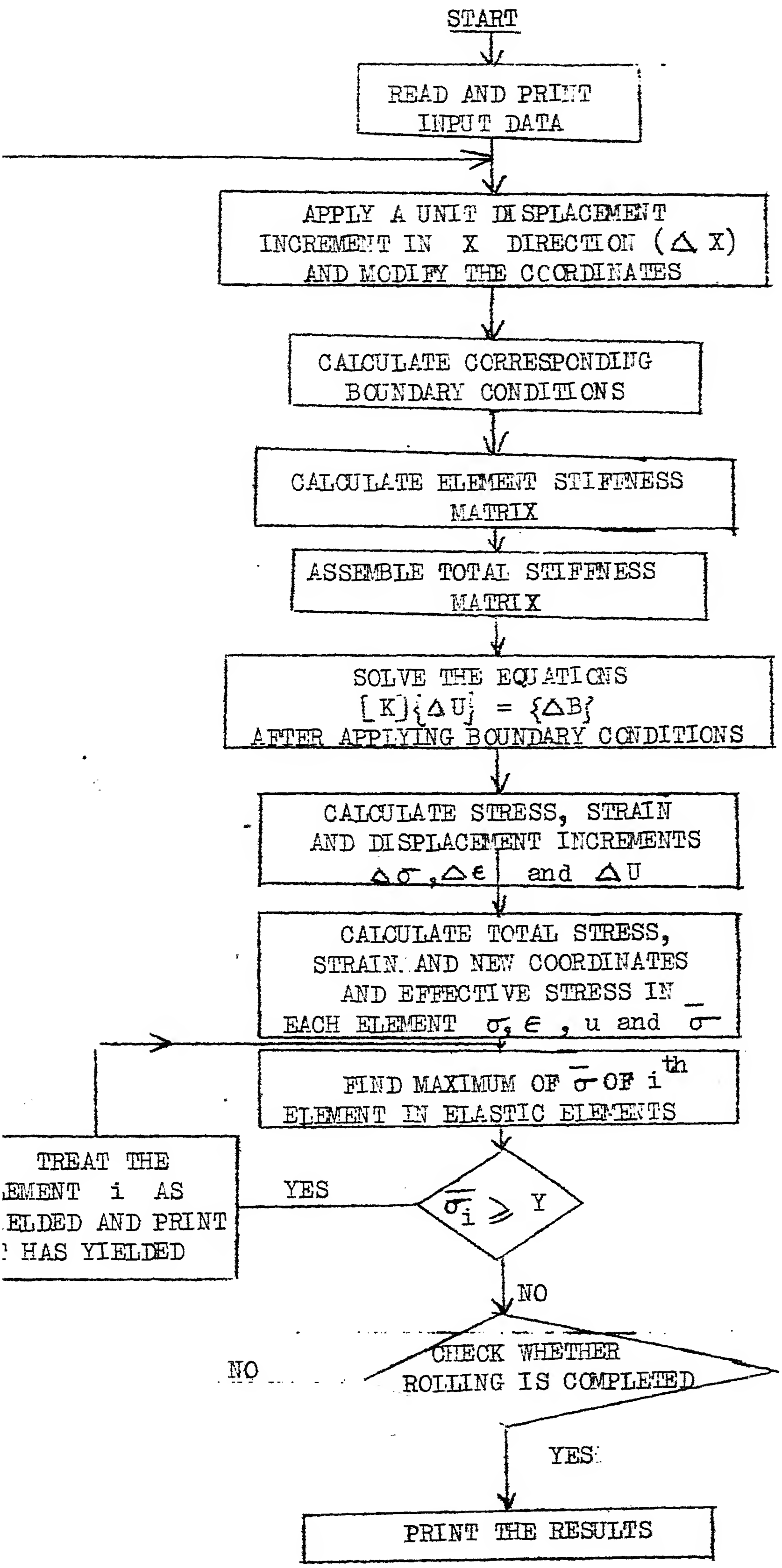

START
READ AND PRINT INPUT DATA
APPLY A UNIT DISPLACEMENT INCREMENT IN X DIRECTION (Δ X) AND MODIFY THE COORDINATES
CALCULATE CORRESPONDING BOUNDARY CONDITIONS
CALCULATE ELEMENT STIFFNESS MATRIX
ASSEMBLE TOTAL STIFFNESS MATRIX
SOLVE THE EQUATIONS [K]{ΔU} = {ΔB} AFTER APPLYING BOUNDARY CONDITIONS
CALCULATE STRESS, STRAIN AND DISPLACEMENT INCREMENTS Δσ, Δε and ΔU
CALCULATE TOTAL STRESS, STRAIN AND NEW COORDINATES AND EFFECTIVE STRESS IN EACH ELEMENT σ, ε, u and σ̄
FIND MAXIMUM OF σ̄ OF i th ELEMENT IN ELASTIC ELEMENTS
σ̄i ≥ Y
YES
TREAT THE
EMENT i AS
ELDED AND PRINT
HAS YIELDED
NO
CHECK WHETHER ROLLING IS COMPLETED
NO
YES
PRINT THE RESULTS

FIG. 5

2.5 NUMERICAL RESULTS

The numerical results obtained in the analysis of cold rolling process are presented here. The material properties taken for this analysis are as follows:

Modulus of elasticity for the material rolled = 1.27×10^4 Tons/sq in.

Yield strength of the material in tension = 47.8 Tons/sq in.

Poisson's ratio = 0.3

The length of the strip analysed is taken as 0.8". It is found that strains and the stresses beyond this length are negligible, when the strip is rolled. The convergence of the results with different number of finite elements is shown in Appendix C. Two initial thicknesses of the strip, namely, 0.064" and 0.063" are considered for the analysis. The final thickness in both the cases is assumed to be 0.0378". The roll radius is taken as 5" in both the cases.

Totally four analyses have been conducted by using 400 elements with 255 nodes. This corresponds to 510 degrees of freedom with a bandwidth of 12 or 14 depending on the orientation of the elements. In the first two cases, the material is taken as elastic-perfectly plastic with an initial thickness of 0.064",

with different orientations of the elements as shown in Table 2. In the next two cases, the orientation of the elements and the initial thickness (0.063")are same, but the behavioral models of the material were different (elastic-perfectly plastic model for the first one and the Ramberg Osgood model for the second one). Table 2 gives the roll forces and torques predicted by the present analysis along with experimental values for the initial thickness 0.063" and final thickness 0.0378" [21] . Figures 7 and 8 show a comparison of the pressure distributions along the roll face obtained for different orientations of the elements and for different models for the elastic-plastic behaviour of the materials, respectively. The figures indicate that the peak values of the pressure on the rolls goes up with an increase in the roll force. Figure 6 gives the deformation pattern and flow of material under the rolls and the locations of elastic and plastic zones for the strip having 0.063" initial thickness and for a material behaving according to Ramberg Osgood model.

Table 1 gives typical results obtained by the finite element analysis for a material having Ramberg Osgood elastic-plastic behaviour for a strip of 0.063" initial thickness. The columns 2 to 5 of this table

show the typical results obtained by the computer program, whose listing is given in Appendix D. This information is used to calculate the total roll force and torque and the pressure distribution along the work-roll interface.

It can be seen from Table 2 that the numerical values of the roll force and torque predicted by the present analysis are higher than the experimental values. The reason for higher values of forces and torques might be due to the "rigid rolls" assumption made in this work. The elasticity of the rolls can be considered to improve the accuracy of the numerical results by using Hitchcock's equation, (3.20). However, this involves the use of present analysis program in an iterative loop. Thus the total analysis time will be multiplied by as many times as the number of iterations required to achieve the desired convergence for the radius of rolls. In the present analysis (with rigid roll assumption), the execution time was about 34 minutes on an IBM 7044 computer. Hence, even if we assume that it takes only five or six iterations to attain the desired convergence, the total time would be about 3 hours.

TABLE 1

Components of Force acting at various nodes along the roll-work interface.

Initial Thickness of strip = 0.063" Output Thickness of strip = 0.0378"

Elastic behaviour of the material ≑ Ramberg-Osgood model

2 Node No. See Fig. (6)	3 F_{x_i} Tons	4 F_{y_i} Tons	5 x_i inch	6 y_i inch	7 Roll Pressure at Node i Tons/sq in.
141	1.2763	-1.4670	0.806136	0.031067	-80.5
146	-0.0776	-1.0555	0.825035	0.029782	-56.8
151	-0.0012	-0.9857	0.843276	0.028610	-53.8
156	0.0484	-0.9979	0.861661	0.027497	-55.0
161	0.281	-1.0900	0.880567	0.026423	-59.0
166	0.0253	-1.1751	0.898623	0.025464	-63.7
171	0.0193	-1.2747	0.917473	0.024533	-67.5
176	-0.00064	-1.2935	0.936393	0.023670	-70.0
181	-0.0312	-1.3320	0.954428	0.022914	-72.3
186	-0.0514	-1.3377	0.973278	0.022194	-71.8
191	-0.0835	-1.2944	0.991660	0.021560	-70.5
196	-0.0992	-1.2004	1.010019	0.020995	-66.7
201	-0.1217	-1.1051	1.027608	0.020517	-61.8
206	-0.1328	-0.9809	1.045805	0.020087	-54.8
211	-0.1485	-0.8268	1.063333	0.019736	-46.8
216	-0.1553	-0.6592	1.081144	0.019442	-37.4
221	-0.1575	-0.4699	1.098578	0.019215	-27.0
226	-0.1518	-0.2845	1.115944	0.019051	-16.5
231	-0.1229	-0.1423	1.133030	0.018947	- 8.35
236	-0.062	+0.0690	1.150053	0.018902	+ 3.50

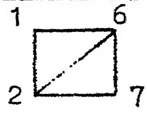
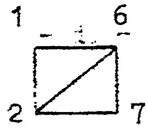
...l roll force* $= \sum_i F_{y_i} = 18.973$ tons

...l roll torque* $= \sum_i F_{x_i} (OQ - y_i) + \sum_i F_{y_i} (OQ - x_i) = 3.978$ Ton-in.

...e Fig. 3 for notation

TABLE 2

Roll forces and torque required to deform the material

S.No.	Elastic-plastic model	Initial Thickness	Final Thickness	Element configu-ration	Roll Force Tons	Torque Ton-in
1.	Elastic Perfectly Plastic Model	0.064"	0.0378"		18.664	3.966
2.	- do -	0.064"	0.0378"		17.077	2.882
3.	- do -	0.063"	0.0378"	- do -	16.782	3.642
4.	Ramberg Osgood model	0.063"	0.0378"	- do -	18.973	3.978
5.	Experimental results [21]				16.56	2.37

CHAPTER 3

OPTIMISATION OF THE COLD ROLLING PROCESS

3.1 INTRODUCTION

The problem of rolling mill design can be formulated as an optimisation problem. Optimisation means either minimisation or maximisation of a function of the design variables and parameters of the system. If there is any restriction on the choice of the design variables, the problem is called a constrained optimisation problem, otherwise it is called an unconstrained optimisation problem. The function, with respect to which the optimisation is carried out is known as the objective function and the restrictions on the design variables due to geometric necessities are called geometric constraints.

A general optimisation problem can be stated in standard form as follows:

Find $\vec{x}$ which minimises $f(\vec{x})$ subject to the constraints.

$$g_j(\vec{x}) \leqslant 0 \qquad j = 1, 2, \ldots m \qquad (3.1)$$

where $\vec{x}$ is called the design vector and consists of the design variables $x_1, x_2, \ldots x_n$, $f(\vec{x})$ is called the objective function and $g_j(\vec{x})$ is called the j^{th} constraint function.

While designing a rolling mill, the designer is confronted with the problem of choosing an appropriate objective function. The objective can be taken as the minimisation of the deflection of rolls or power required to drive the mill. The minimisation of the deflection of rolls has to be taken if one wants better quality product, while the minimisation of power is to be chosen for bringing down the cost of production.

Since both the objectives can not be achieved simultaneously, one is used as a constraint and the other is treated as the objective in the present work. Thus when deflection is taken as the objective the minimisation is carried after deciding about the motors available and hence their output power puts an upper bound on the power required for the rolling process. In the second case, the minimisation of the power required for rolling is carried by placing an upper bound on the maximum possible deflection of the rolls.

In order to evaluate the objective functions and the constraints, the expressions for the roll force and torque are required in terms of the roll dimensions, the exit and the entry tensions and the dimensions of the material being rolled. These expressions are derived in the following sections.

3.2 EXPRESSIONS FOR THE OBJECTIVE AND CONSTRAINT FUNCTIONS

In practice, for cold rolling, the strip has to pass through three zones. In the first zone, i.e., near the plane of entry to the roll gap, the strip is elastically compressed until the yield stress is reached, in the second zone it is further compressed plastically and in the third zone it recovers elastically, Fig. (9).

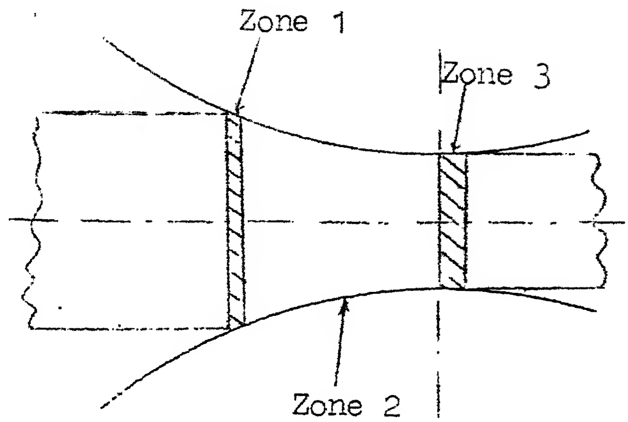


FIG. (9)

3.2.1 Plastic Deformation of a Strip

By considering the force balance of a transverse slice of material rolled, shown in Fig. (1), one gets the basic differential equation.

$$\frac{d\,f}{d\,\phi} = 2\gamma R\,(\sin\phi \mp \mu \cos\phi) \qquad (3.2)$$

Bland and Ford [6] made the following assumptions to derive the normal roll pressure from this basic differential equation for the rolling process.

(1) The arc of contact remains circular even when roll flattening occurs.

(2) μ is constant for the whole length of the arc of contact.

(3) The elastic compression is negligible (This assumption is waved later when the elastic deformation is also taken into account).

(4) The spread is zero.

(5) Von-Mises criterion of plasticity is employed.

(6) The deformation is wholly homogeneous.

(7) The radial pressure 's' at any point along the arc of contact is equal to its vertical component.

The normal roll pressure is given by [6] :

$$s^{+} = \frac{k\,h}{h_2}\left(1 - \frac{\sigma_2}{k_2}\right) e^{\mu H} \qquad (3.3)$$

from exit to the neutral point and

$$s^{-} = \frac{k\,h}{h_1}\left(1 - \frac{\sigma_1}{k_1}\right) e^{\mu(H_1 - H)} \qquad (3.4)$$

from entry to the neutral point, where

$$H = 2\sqrt{\frac{R'}{h_2}}\,\tan^{-1}\left(\sqrt{\frac{R'}{h_2}}\,\phi\right) \qquad (3.5)$$

and

$$H_1 = 2\sqrt{\frac{R'}{h_2}}\,\tan^{-1}\left(\sqrt{\frac{R'}{h_2}}\,\phi_1'\right) \qquad (3.6)$$

ϕ_1 is given by

$$\phi_1 = \sqrt{\frac{(h_1 - h_2)}{R'}} \tag{3.7}$$

The roll force is obtained from the expression

$$P = R' \int_0^{\phi_n} s^+ \, d\phi + R' \int_{\phi_n}^{\phi_1} s^- \, d\phi \tag{3.8}$$

where ϕ_n, the angle of the neutral point is given by

$$\phi_n = \sqrt{\frac{h_2}{R'}} \tan \left(\sqrt{\frac{h_2}{R'}} \cdot \frac{H_n}{2} \right) \tag{3.9}$$

$$\text{and} \quad H_n = \frac{H_1}{2} - \frac{1}{2a} \log_e \left\{ \left(\frac{h_1}{h_2} \right) \left(\frac{1 - \sigma_2 / k_2}{1 - \sigma_1 / k_1} \right) \right\} \tag{3.10}$$

The roll torque is given by the expression

$$G = RR' \left[\int_0^{\phi_n} s^+ \phi \, d\phi + \int_{\phi_n}^{\phi_1} s^- \phi \, d\phi + \frac{(\sigma_1 h_1 - \sigma_2 h_2)}{2 R'} \right] \tag{3.11}$$

In these equations, the tensions σ_1 and σ_2 denote the absolute values, the signs having been taken care of in the derivation.

The problem in evaluating P from equation (3.8) lies in the fact that the yield stress k occurs as a variable in equations (3.3) and (3.4). Ford and Bland [6] has shown that when mean yield stress is used in place of 'k' as an approximation the error will not be more than 2% for cold rolling.

When the mean yield stress $\bar{k}$ is taken as a constant, the expressions for the roll force and the torque can be given in terms of certain non-dimensional quantities, as --

$$P = \bar{k} R' (h_1 - h_2) \cdot (1 - \frac{\sigma_1}{\bar{k}}) \cdot f_3 (a, r, b) \quad (3.12)$$

and

$$G = R \bar{k} (h_1 - h_2) \cdot (1 - \frac{\sigma_1}{\bar{k}}) \cdot f_5 (a, r, b) \quad (3.13)$$

where non-dimensional parameters are given by

$$a = \mu \sqrt{\frac{R}{h_2}}$$

$$b = (1 - \frac{\sigma_2}{\bar{k}}) / (1 - \frac{\sigma_1}{\bar{k}})$$

and $r = (h_1 - h_2) / h_1$

The functions f_3 and f_5 and the derivation of the expressions for P and G are given in Appendix (B).

3.2.2 Elastic Deformation of a Strip

The effect of elastic deformation in cold rolling of strip is three fold, the total roll force is thereby directly augmented; the friction between the roll and the strip over the elastic arc increases the "friction hill" there by increasing the plastic roll force; and the effective roll radius is modified.

Except for very small passes, the elastic compression at the entry causes negligible difference in the roll force. The contribution of the elastic effect at exit, however cannot be neglected; in small passes it may account to 15% of the total roll force, and in normal reductions, it is seldom less than 4% [21].

Making certain assumptions and approximations, Ford Ellis and Bland [22] have shown that the elastic contribution to the roll force is given by:

At entry:

$$P_{e_1} = \frac{(1 - \nu^2)\, h_1}{4} \sqrt{\frac{R'}{h_1 - h_2}} \left(\frac{k - \sigma_1}{E} \right)^2 \qquad (3.14)$$

At exit:

$$P_{e_2} = \frac{2}{3} (k - \sigma_2) \sqrt{\frac{h_2\, R'}{E} (1 - \nu^2)\, (k - \sigma_2)} \qquad (3.15)$$

The contribution to roll torque are given by

$$\begin{aligned} G_{e_1} &= \mu R\, P_{e_1} \\ G_{e_2} &= -\mu R\, P_{e_2} \end{aligned} \qquad (3.16)$$

The new effective back tension is

$$\sigma_1' = \sigma_1 - \frac{2\mu P_{e_1}}{h_1} \qquad (3.17)$$

and the effective front tension at the beginning of the plastic zone is

$$\sigma_2' = \sigma_2 - \frac{2\mu P_{e_2}}{h_2} \qquad (3.18)$$

If h_m is the minimum thickness of the strip in the pass, it is given by

$$h_m = h_2 - \nu \frac{(1+\nu)}{E} h_2 \sigma_2 + \frac{h_2 (1-\nu^2)(k - \sigma_2)}{E} \qquad (3.19)$$

The Hitchcock's formula for the radius of the contact arc R', namely,

$$R' = R\left(1 + \frac{16(1-\nu^2)}{\pi E} \cdot \frac{P}{h_1 - h_2}\right) \qquad (3.20)$$

is modified, due to the presence of elastic zone,

$$R' = R\left(1 + \frac{16(1-\nu^2)}{\pi E} \cdot \frac{P}{(\sqrt{\delta + \delta_2 + \delta_t} + \sqrt{\delta_2})^2}\right) \qquad (3.21)$$

where

$$\delta = h_1 - h_2$$

$$\delta_2 = \frac{h_2(1-\nu^2)}{E}(k_2 - \sigma_2)$$

and $\delta_t = \frac{\nu(1-\nu)}{E}(h_2 \sigma_2 - h_1 \sigma_1)$

Thus the total roll force and the roll torque, by considering the elastic deformation,

are given by

$$P = P_p + P_{e_1} + P_{e_2} \qquad (3.22)$$

and $$G = G_p + G_{e_1} + G_{e_2} \qquad (3.23)$$

where P_p and G_p are roll force and torque due to plastic deformation of the strip.

To calculate the force and the torque due to plastic deformation, we use the new effective back and front tensions given by equation (3.17) and (3.18). Also we use h_m, given by equation (3.19), instead of h_2 as the plastic zone spreads upto the minimum thickness between the rolls, beyond which the elastic recovery starts.

To calculate the exact value of the roll force and the torque an iterative procedure is used. The equations for the roll force and the torque involve the modified radius R'. When calculation is started R' is assumed to be equal to R. Once the roll force and the torque are calculated, the calculated value of the roll force is used to find R' from modified Hitchcock's formula (3.21). The new value of R' is now used to calculate a more accurate roll force and torque. The procedure is continued till the desired accuracy for the roll force or the roll radius or for the both is achieved.

3.2.3 Permissible Loads for Rolls

Once the torque required to roll the material and the roll force are known, the stresses present in the rolls can be calculated. The upper bound on these stresses is going to a constraint in the optimisation problem.

With respect to Fig. (10), Wusatowski [23] gives expressions for the bending stresses, under the action of roll forces and the torsional stresses due to roll torque.

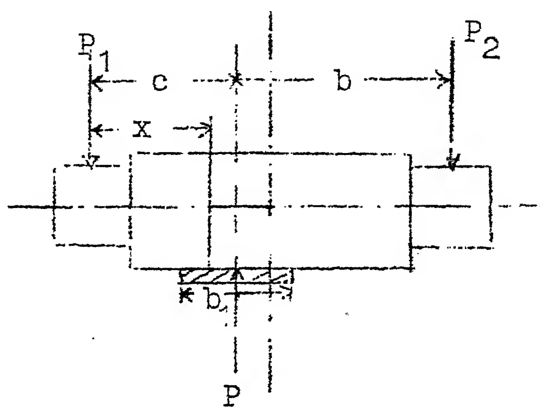


FIG. (10)

The bending stress is given by

$$\sigma_b = \frac{32}{\pi D^3} b_1 P \left(\frac{bC}{L_1} - \frac{b_1}{8} \right) \tag{3.24}$$

where $L_1 = b + C$.

If the material is rolled in the centre of rolls, which is the case assumed in the present work,

$$\sigma_b = \frac{b_1 P}{\pi R^3} \left(L_1 - \frac{b_1}{2} \right) \tag{3.25}$$

The torsional stress in the roll barrel is given by

$$\tau = \frac{2 G b_1}{\pi R^3} \tag{3.26}$$

The expression for the compound stress under the simultaneous action of bending and torsion is given by

$$\sigma_b' = \sqrt{\sigma_b^2 + 3\tau^2} \quad \text{for steel rolls} \tag{3.27}$$

and $$\sigma_b' = .375\,\sigma_b + .625\sqrt{\sigma_b^2 + 4\tau^2} \quad \text{for cast-iron rolls.} \tag{3.28}$$

3.2.4 Deflection of Rolls

Once the roll force is known the extent of deflection at any point from the end of barrel to midway along the length of the roll can be found from the equations

$$C_{yB} = \frac{P\,(x - n)}{24\,E\,I}\left[\frac{(x - n)^3}{b_1} + 4\,b^2 + 2\,b_1\,n - 2x\,(x + n)\right] \tag{3.29}$$

and $$C_{yS} = \frac{5\,P}{\pi\,E\,D^2}\left[\frac{x\,(2b - x) - n\,(2b - n)}{b_1}\right] \tag{3.30}$$

where $n = b - \frac{b_1}{2}$,

the meaning of the other symbols is indicated in Fig. (10). The deflection at the ends of the roll barrel (at $x = n$, and $x = 2b - n$) is equal to zero. Hence maximal deflection will occur at $x = b = L_1/2$.

As suffix B denotes the deflection due to bending forces and S due to shearing forces, the total

deflection of rolls is given by

$$C_y = C_{yB} + C_{yS} \qquad (3.31)$$

3.2.5 Energy Consumed in Cold Rolling

Broadly speaking, the total energy required for rolling can be divided into four parts:

(1) the energy needed to deform the material as it passes through the rolls, called basic energy of rolling,

(2) the energy needed to overcome the frictional forces in the roll neck bearing,

(3) the energy required to balance the frictional losses in pinions, reduction gear etc.,

(4) the energy required to compensate for losses in the various electric motors, generators and the complete electrical circuit.

Further when significant amounts of coiler, decoiler tensions are employed, some energy would be needed to pull the strip through the rolls. In this work, the energy consumed in rolling is computed by considering parts (1) and (2) alongwith the energy due to coiler and decoiler tensions.

The basic energy of rolls is nothing but the work done in overcoming the torque required to roll the material, where

work done per roll $= 2\pi G$ per revolution.

Hence basic energy for two rolls $= 4\pi G$ per revolution.

The work required to overcome the frictional losses in the roll neck bearings of two rolls $= 2 \times 2\pi \frac{P}{2} \mu' d$

$$= 2\pi P \mu' d$$

The coiler and decoiler energy consumption per minute is given by the following expressions:

Energy consumed per minute (coiler) $= F\, v$

Energy consumed per minute (decoiler) $= B\, v \frac{h_2}{h_1}$

where F is the pull and B is the drag and v is the speed of strip being coiled on the coiler.

If b_1 denotes the width of the strip,

$$\sigma_2 = \frac{F}{h_2\, b_1}$$

$$\sigma_1 = \frac{B}{h_1\, b_1}$$

Hence,

Energy consumed per minute (coiler) $= \sigma_2\, h_2\, b_1 \cdot v$

Energy consumed per minute (decoiler) $= \sigma_1\, h_2\, b_1 \cdot v$

Total energy consumed by coiler and decoiler

$$= (\sigma_1 + \sigma_2) \frac{h_2 b_1 v}{N} \text{ per revolution}$$

where, N is the number of revolutions per minute of the rolls.

Total energy consumed per revolution for strip rolling in the mill

$$= 4\pi G + 2\pi P\mu' d + (\sigma_1 + \sigma_2) \frac{h_2 b v}{N}$$

Here $\frac{h_2 b v}{N}$ can be expressed in terms of the other dimensions of the roll as

$$\frac{h_2 b v}{N} = 2\pi R \left[1 + \frac{2R}{h_2} (1 - \cos \phi_n) \right] \quad (3.32)$$

where, ϕ_n is the neutral angle and is given by Eq. (3.9).

Therefore, total energy consumption in cold rolling of strip in the mill

$$= 2\pi \left[2G + P\mu' d + R (\sigma_1 + \sigma_2) \left(1 + \frac{2R}{h_2} (1 - \cos \phi_n)\right) \right] \quad (3.33)$$

3.3 CONSTRAINT SET

For the optimisation of the objective functions considered, the factors that impose restrictions on the design parameters need to be considered. The

restrictions come from various considerations like the stresses developed in the rolls, the location of the neutral point, the angle of bite and the deflection of rolls / the required power. The following constraints are considered in the present formulation:

(1) The lower bound on the radius of rolls is taken as zero. No upper bound is considered on the radius of the rolls as the power restriction (either as a constraint or as an objective) limits its value.

(2) The lower bound on the back and front tensions is taken as zero. The upper bound is not considered because of the same reason as given in (1).

(3) If the neutral angle, ϕ_n, is less than zero, the rolls will skid over the material and if it is greater than the angle of bite, ϕ_1; the rolls jam and will not move. Hence ϕ_n is restricted as

$$0 \leqslant \phi_n \leqslant \phi_1$$

(4) The angle of bite, ϕ_1, is constrained to be positive as

$$\phi_1 \geqslant 0$$

The maximum angle of bite is dictated by the

friction between the rolls and the material. If the angle of bite is more than twice the angle of friction, we need to apply front tension. Hence, ϕ_1 is limited as

$$\phi_1 \leqslant 2 \tan^{-1}(\mu)$$

(5) The compound stress in the rolls due to bending and shearing forces should not be more than a certain permissible value, which is dictated by the material of the rolls:

$$\sigma_b' \leqslant \sigma_{permissible} \, .$$

(6) When minimization of the deflection of rolls is taken as the objective, the power required for the rolling process is restricted as:

$$2\pi\left[\left(2G + P\mu' d + R\,(\sigma_1 + \sigma_2)(1 + \frac{2R}{h_2}(1 - \cos \phi_n)\right)\right] \leqslant P_{max}$$

where P_{max} is the minimum power available.

If minimization of power is taken as the objective, the deflection of rolls is limited to a maximum value of δ_{max} as:

$$C_y = C_{yB} + C_{yS} \leqslant \delta_{max}$$

where δ_{max} is the maximum permissible deflection, which will be decided by the quality requirements of the rolled product.

3.4 STATEMENT OF THE OPTIMISATION PROBLEM FOR A SINGLE PASS ROLLING

For a single pass rolling, the design vector is taken as

$$\vec{X} = \begin{bmatrix} R \\ \sigma_1 \\ \sigma_2 \end{bmatrix} \qquad (3.34)$$

The expressions for the objective function is given by

$$f_1(\vec{X}) = C_y = C_{yB} + C_{yS} \qquad (3.35)$$

where C_{yB} and C_{yS} are given by equations (3.29) and (3.30) respectively, if deflection is taken as the objective,

or $$f_2(\vec{X}) = 2\pi\Big[2G + P\mu' d + R(\sigma_1 + \sigma_2) \cdot \left(1 + \frac{2R}{h_2}(1 - \cos\phi_n)\right)\Big] \qquad (3.36)$$

if power is taken as the objective.

The nine constraints can be stated as:

(1) $-R \leqslant 0$ (3.37)

(2) $-\sigma_1 \leqslant 0$ (3.38)

(3) $-\sigma_2 \leqslant 0$ (3.39)

(4) $-\phi_n \leqslant 0$ (3.40)

where ϕ_n is given by Eq. (3.9)

(5) $\phi_n - \phi_1 \leqslant 0$ (3.41)

where ϕ_1 is given by Eq. (3.7)

(6) $-\phi_1 \leqslant 0$ (3.42)

(7) $\phi_1 - 2\tan^{-1}(\mu) \leqslant 0$ (3.43)

(8) $\sigma_b - \sigma_{permissible} \leqslant 0$ (3.43a)

where σ_b is given by Eq. (3.27)

(9)
$$2\pi\left[2G + P\mu' d + R(\sigma_1 + \sigma_2) \cdot \left(1 + \frac{2R}{h_2}(1 - \cos\phi_n)\right)\right] - P_{max} \leqslant 0 \quad (3.44)$$

when deflection is taken as objective,

or $(C_{yB} + C_{yS}) - \delta_{max} \leqslant 0$ (3.45)

where C_{yB} and C_{yS} are given by equations (3.29), (3.30) respectively, when power is taken as objective.

3.5 STATEMENT OF THE OPTIMISATION PROBLEM FOR MULTIPASS ROLLING IN TANDEM MILL

In the case of material being rolled in a tandem mill having m single pass rolling mills in series, the objective function is taken as the sum of deflections of the rolls in all the 'm' rolling mills.

Hence the objective function in tandem mill rolling is given by:

$$f(\vec{x}) = \sum_{i=1}^{m} (C_{yB_i} + C_{yS_i}) \qquad (3.46)$$

where C_{yB_i} and C_{yS_i} are the deflections due to bending and shear stresses in the i^{th} mill and are given by equations (3.29), (3.30) respectively.

The design vector, $\vec{x}$, is taken as

$$\vec{x} = \begin{bmatrix} R_1 \\ \vdots \\ R_m \\ \sigma_1 \\ \vdots \\ \sigma_{m+1} \\ H_2 \\ \vdots \\ H_m \end{bmatrix} \qquad (3.47)$$

where R_i is the radius of the rolls in the i^{th} mill, σ_1 is the back tension in the first mill, σ_{m+1} is the front tension of the last (m^{th}) mill, σ_i is the tension between $i - 1^{th}$ and i^{th} mills ($i = 2, 3, \ldots m$), H_i is the thickness of the strip between $i - 1^{th}$ and i^{th} mills ($i = 2, 3, \ldots m$), H_1 is the input thickness and H_{m+1} is the output thickness of the strip.

The constraints can be expressed as:

(1) $- R_i \leqslant 0$ $\quad i = 1, \ldots m$ (3.48)

(2) $- \sigma_i \leqslant 0$ $\quad i = 1, \ldots m, m + 1$ (3.49)

(3) $- \phi_{ni} \leqslant 0$ $\quad i = 1, \ldots m$ (3.50)

where ϕ_{ni} is the neutral angle in the i^{th} mill and is given by Eq. (3.9).

(4) $\phi_{ni} - \phi_{1i} \leqslant 0$ $\quad i = 1, \ldots m$ (3.51)

where ϕ_{1i} is the angle of bite in the i^{th} mill and is given by Eq. (3.7).

(5) $- \phi_{1i} \leqslant 0$ $\quad i = 1, \ldots m$ (3.52)

(6) $\phi_{1i} - 2 \tan^{-1} (\mu_i) \leqslant 0$ $\quad i = 1, \ldots m$ (3.53)

where μ_i is the friction between material and rolls in the i^{th} mill.

(7) $\sigma'_{b_i} - \sigma_{i\,permissible} \leqslant 0$ $\quad i = 1, \ldots m$ (3.54)

where $\sigma_{i\,permissible}$ is maximum permissible stress for the rolls, and σ'_{b_i} is the total stress on the rolls in rolling in the i^{th} mill and is given by Eq. (3.27).

(8) $$2\pi \left[2\, G_i + P_i \mu'_i\, d_i + R_i \, (\sigma_i + \sigma_{i+1}) \cdot (1 + \frac{2\, R_i}{h_i + 1} (1 - \cos \phi_n)) \right] - (P_{max})_i \leqslant 0$$

$i = 1, \ldots m$ (3.55)

3.6 SOLUTION TECHNIQUE

The optimisation problem formulated in sections (3.4) and (3.5) can be seen to be of the form given by equation (3.1). Keeping in mind that an unconstrained optimisation problem is always easier to solve compared to a constrained problem, the method of Sequential Unconstrained Minimisation Technique (SUMT) has been applied to solve the constrained minimisation problems formulated earlier. The steps involved in finding the solution by SUMT method are as follows:

<u>Step 1</u>

Determine an initial feasible vector $\vec{x}_o$, which will satisfy all the constraints. finding $\vec{x}_o$, care has to be taken to see that it will be a free vector but not a bound vector. A free vector is a point which will satisfy all the constraints with strict inequality and a bound vector is one which will satisfy at least one constraint with equality. The reason for avoiding a bound vector is that if one or more constraints are satisfied with strict equality sign, the term $\sum_{j=1}^{m} \frac{1}{g_j(\vec{x})}$, which will be needed in the subsequent steps, will become infinite.

Step 2

Construct a new function ϕ as

$$\phi(\vec{x}, r) = f(\vec{x}) - r \sum_{j=1}^{m} \frac{1}{g_j(\vec{x})} \quad (3.57)$$

where r is some positive constant.

In the present work, the starting value of r is chosen so that the two terms in the expression for ϕ are approximately equal when evaluated at the point $\vec{x}_o$.

$$\therefore \text{Starting value of } r = r_o = - \frac{f(\vec{x}_o)}{\sum_{j=1}^{m} \frac{1}{g_j(\vec{x}_o)}} \quad (3.58)$$

Step 3

Starting from the vector $\vec{x}_o$, find the unconstrained minimum of $\phi(\vec{x})$ for a fixed value of $r = r_o$. The Davidon Fletcher Powell (DFP) method is used to minimise the ϕ function.

This method involves the following steps:

(1) Start with a given vector $\vec{x}_o$ and an initial positive definite symmetric matrix $[H_o]$. $[H_o]$ can be taken as an identity matrix.

Then set $S_o = -[H_o] \nabla \phi_o$ (3.59)

where $\nabla \phi_o$ is the gradient vector of the function ϕ evaluated at $\vec{x}_o$ and $\vec{S}_o$ is called the direction vector. Set the index $q = 0$ for generalizing the procedure.

(ii) find a new vector $\overline{x}_{q+1}$ as

$$\vec{x}_{q+1} = \vec{x}_q + \alpha_q^* \vec{S}_q \tag{3.60}$$

where α_q^* minimises $\phi(\vec{x}_q + \alpha_q^* \vec{S}_q)$. This α_q^* is known as the optimum step length along the direction $\vec{S}_q$ and step (4) gives a procedure for finding α_q^*.

(iii) Evaluate $\nabla \phi_{q+1}$, that is, the gradient vector of ϕ at $\vec{x}_{q+1}$. Generate the new search direction as

$$\vec{S}_{q+1} = -\left[H_{q+1}\right] \nabla \phi_{q+1} \tag{3.70}$$

where $\left[H_{q+1}\right] = \left[H_q\right] + \left[M_q\right] + \left[N_q\right]$

with $$\left[M_q\right] = \alpha_q^* \frac{\vec{S}_q \cdot \vec{S}_q^{\,T}}{\vec{S}_q^{\,T} \cdot \vec{Y}_q}$$

$$\left[Y_q\right] = \nabla \phi_{q+1} - \nabla \phi_q$$

and $$\left[N_q\right] = - \frac{(\left[H_q\right] \vec{Y}_q (\left[H_q\right] \vec{Y}_q)^T}{\vec{Y}_q^{\,T} \left[H_q\right] \vec{Y}_q}$$

(iv) Once the new search direction $\vec{S}_{q+1}$ is known, set $q = q + 1$ and repeat steps (ii) through (iv) untill the function $\emptyset$ is minimised. To find whether the function $\emptyset$ is minimised in any particular iteration q, a convergence criterion has to be used at the end of step (ii).

Step 4

This step indicates a method for determining the optimum step length α_q^*, from a given point and along a particular search direction. The cubic interpolation techniques has been applied for this purpose.

α_q^* is the quantity which minimises $\emptyset(\vec{x}_q + \alpha_q \vec{S}_q)$, where $\vec{x}_q$ and $\vec{S}_q$ are already known. Hence $\emptyset(\vec{x}_q + \alpha_q \vec{S}_q)$ can be treated as a function of α_q alone and it is known as the one dimensional minimisation problem.

When subscript q is dropped, let

$$f(\alpha) = \emptyset(\vec{x} + \alpha \vec{S}) \tag{3.71}$$

Approximating $f(\alpha)$ by a cubic polynomial $h(\alpha)$, $f(\alpha)$ can be written as

$$f(\alpha) \simeq h(\alpha) = a + b\alpha + c\alpha^2 + d\alpha^3 \tag{3.72}$$

Let for $\alpha = A = 0$ $\quad f = f_A$ and $f' = f_A' < 0$

and for $\alpha = B$ $\quad f = f_B$ and $f' = f_B' > 0$

Then the value of α which minimises $h(\alpha)$ is given by

$$\overline{\alpha^*} = A + \frac{f_A' + Z + Q}{f_A' + f_B' + 2Z} (B - A) \tag{3.73}$$

where $Z = \dfrac{3(f_A - f_B)}{(B - A)} + f_A' + f_B'$

and $\quad Q = (Z^2 - f_A' f_B')^{1/2}$

After finding $\overline{\alpha^*}$, convergence criterion

$$\frac{|\vec{V}^T \cdot \vec{S}|}{|\vec{V}| \cdot |\vec{S}|} \leqslant \epsilon \tag{3.74}$$

where $V = \nabla \phi \big|_{\overline{\alpha^*}}$, should be checked to see whether * also corresponds to minimum of $f(\alpha)$. If the convergence criterion is satisfied, $\overline{\alpha}^*$ can be taken as α^*, otherwise the procedure has to be repeated as stated below.

Evaluate $\dfrac{df}{d\alpha}$ at $\overline{\alpha^*}$. It will be either positive or negative.

(i) If $\dfrac{df}{d\alpha}\Big|_{\overline{\alpha^*}}$ is positive, set $B = \overline{\alpha^*}$, $f_B = f(\overline{\alpha^*})$ and $f_B' = \dfrac{df}{d\alpha}\Big|_{\overline{\alpha^*}}$ and calculate a new value of $\overline{\alpha^*}$ using equation (3.73).

(ii) If $\frac{df}{d\alpha}\Big|_{\overline{\alpha}^*}$ is negative, set $A = \overline{\alpha}^*$, $f_A = f(\overline{\alpha}^*)$ and $f_A' = \frac{df}{d\alpha}\Big|_{\overline{\alpha}^*}$ and calculate a new value of $\overline{\alpha}^*$ as per equation (3.73).

Once a new value of $\overline{\alpha}^*$ is found, test for convergence according to equation (3.74) and the subsequent procedure has already been stated above.

The steps 1 to 4 indicate how the function ϕ is minimised for a particular value of r. At the end of the minimisation process, a new vector, which minimises $\phi(\vec{x}, r)$, will be obtained. Now the complete procedure is repeated with this new vector as the starting point and with a value of r smaller than the previous value. The process is continued for several values of r and can be terminated when some or all the following conditions are satisfied simultaneously.

(1) Final value of r is very small as compared to r_o.

(2) ϕ_{min} for two consequtive r's are almost equal.

(3) The value of f corresponding to ϕ_{min} for two consequtive r's are almost equal.

(4) There is no significant difference between the values of ϕ and f.

(5) The design vector corresponding to ϕ_{min} for two consequtive r's remains almost identical.

3.7 NUMERICAL RESULTS FOR SINGLE PASS ROLLING

In the present work, cast steel is taken as the material of rolls and 0.08% carbon steel is taken as the material to be rolled in the rolling process.

Numerical Data:

Maximum permissible stress in the rolls = 637 tons/sq in.

Modulus of elasticity of the roll material = 1.275×10^4 tons/sq in.

Coefficient of friction for roll neck bearing = 0.08

Constant in Hitchcock's equation (c) = 1.67×10^{-4}

Modulus of elasticity of the material to be rolled = 1.27×10^4 tons/sq in.

Poisson's ratio for the material to be rolled = 0.3

Coefficient of friction between the roll and the material = 0.055

Average yield strength of the material to be rolled = 37.6 tons/sq in.

The actual yield strengths used for different reductions are given in Table 3 [21].

TABLE 3

Actual yield strength for different reductions (for 0.08% C steel)

% Reduction	0 %	10%	20%	30%	40%	50%	60%	70%
Yield strength (Tons/sq in.)	12.2	30.5	37.0	41.2	44.0	46.2	48.0	49.0

3.7.1 Deflection of Rolls as Objective for Minimisation

The design optimisation problem is solved for three input thicknesses (0.13", 0.063", 0.013") with four percentage reductions for each thickness (10%, 20%, 30%, 40%). Each of these twelve problems is solved by using three different upper bounds on the required power. The numerical results are given in Tables 4 to 6. The following trends can be observed from these tables.

(i) The stresses induced in the rolls have been found to be very small, because in rolling of thin strips, forces exerted on the rolls are small, and are of the order of 17 tons in the case of strips of 0.063" initial thickness having 40% reduction.

(ii) The constraint on power has been observed to be active in all the problems.

(iii) For very thin (0.013") strips, the back and the front tensions at the optimum point are considerably higher compared to those of thicker strips (0.063", 0.013").

(iv) As the percentage reduction increased, the radius of rolls slightly decreased and the tensions increased in the case of very thin strips. In the case of thicker strips, all the three

design variables are reduced as the percentage reduction is increased.

(v) In strip rolling the power required to deform the material is supplied through the torque applied across the rolls and the back and front tensions. In general the power available will be distributed between the torque applied through rolls and the tensions. It has been observed that in the case of very thin strips, an increase in the available power (because of a higher value of P_{max}) results in increased tensions and the radius of the rolls do not change much (rather decreases slightly). This means that the work done by tensions is more and the torque on the rolls will be less, which also implies that roll forces are reduced resulting in a smaller deflection of the rolls.

(vi) In the case of thicker strips, the optimum tensions do not change considerably but the roll radius increases which implies that more power is required for rolling. With an increase in the roll radius, the stiffness of the rolls increases resulting in a smaller deflection of the rolls.

(vii) In Figures 11 to 13, the behavior of the objective function is shown plotted against the percentage reduction for different values of the maximum

power availabe to the rolling mill. It can be observed from these figures that for very thin strips, the availability of more power has a negligible effect on the objective function. However, in the case of thicker strips, the availability of more power results in much greater reductions in the deflection of rolls.

3.7.2 Power Required in Rolling as The Objective For Minimisation

In this case, the design optimisation problem is solved for one input thickness (0.063") with four percentage reductions, namely, 10%, 20%, 30% and 40%. The deflection of rolls is taken as one of the constraints for the problem and an upper limit of 5.0×10^{-4} inch is put on it. The results are given in Table 7. The trends observed in this case are as follows:

(i) The stresses in this case also are very small, because of small roll forces.

(ii) The lower limits on the back and front tensions became the active constraints at the optimum point. In fact, the back and fron tensions are too small for all practical purposes.

(iii) It can be seen that a more efficient way of reducing the power is to decrease the radius of rolls, i.e., to reduce the torque applied through the rolls.

3.8 NUMERICAL RESULTS FOR MULTIPASS ROLLING IN TANDEM MILL

The material properties for the rolls and the material rolled are same as those are given in section 3.7. The problem is solved for a tandem mill having two single pass rolling mills in series. The maximum reductions allowable in the first and the second mill are 40% and 30% respectively. The maximum allowable powers for both the mills are same. It is 800 ton-in per revolution for both the mills. The total reduction given in both the passes is 50% and the initial thickness of the strip is .13".

The initial and the final design vectors are given below:

$$\left\{\vec{X}_o\right\} = \begin{Bmatrix} 4.00000 \\ 4.00000 \\ 0.05000 \\ 0.05000 \\ 0.05000 \\ 0.085 \end{Bmatrix}, \quad \left\{\vec{X}_{opt}\right\} = \begin{Bmatrix} 4.00062 \\ 4.00045 \\ 0.05003 \\ 0.05003 \\ 0.05001 \\ 0.078036 \end{Bmatrix}$$

Objective function at the start of optimisation $= .5253 \times 10^{-2}$ in.

Optimal objective function $= .5055 \times 10^{-2}$ in.

The total number of one-directional optimisation steps are 21 and the total computer time taken is 58 minutes on IBM 7044 computer. The observations are:

i) the tensions and the roll radii are almost same at the optimum point as at the starting point, and

ii) the first mill works to its highest capacity, that is the maximum possible reduction has been obtained in the first pass itself.

The constraint which became active at the optimum point was the upper limit on the maximum reduction allowable in the first pass.

TABLE (4)

Initial Thickness of the Strip Being Rolled = .13"

% Reduction	P_{max} Ton-in/rev.	Design Vector	X_1 Inch.	X_2 Ton/Sq. in.	X_3 Ton/Sq. in.	Obj. fn. (deflection) 10^{-4} in.	Power used Ton-in/rev.	No. of Iterations	Computer time Min : Sec
10%	800.00	Initial	3.000	0.384	0.427	4.372	-	25	09 : 52
		Optimal	8.102	1.024	1.104	2.992	781.04		
20%	800.00	Initial	3.000	0.385	0.481	9.832	-	11	05 : 09
		Optimal	6.239	0.622	0.614	7.459	794.518		
30%	800.00	Initial	3.500	0.385	0.550	42.33	-	23	05 : 01
		Optimal	5.199	0.253	0.278	13.94	794.32		
40%	800.00	Initial	4.000	0.385	0.641	31.260	-	14	05 : 01
		Optimal	4.419	0.463	0.644	23.660	795.49		
10%	900.00	Initial	8.063	0.429	0.293	3.032	-	34	11.: [illegible]
		Optimal	8.711	0.436	0.081	2.565	854.2		
20%	900.00	Initial	6.239	0.622	0.614	7.459	-	26	04 : 56
		Optimal	6.604	0.531	0.402	6.498	892.14		
30%	900.00	Initial	5.199	0.253	0.278	13.94	-	23	05 : 11
		Optimal	5.638	0.370	0.148	11.300	887.16		
40%	900.00	Initial	4.419	0.463	0.644	23.660	-	30	10 : 14
		Optimal	4.907	0.233	0.352	17.833	897.19		
10%	1000.00	Initial	8.711	0.436	0.081	2.565	-	17	14 : 00
		Optimal	8.716	0.631	0.078	2.104	972.6		
20%	1000.00	Initial	6.604	0.531	0.402	6.498	-	12	05 : 06
		Optimal	7.304	0.555	0.977	5.124	996.32		
30%	1000.00	Initial	5.638	0.370	0.148	11.300	-	15	05 : 06
		Optimal	5.987	0.853	0.590	9.698	977.09		
40%	1000.00	Initial	4.907	0.233	0.352	17.833	-	22	07 : 11
		Optimal	5.285	0.368	0.381	14.674	993.83		

TABLE (5)

Initial Thickness of the Strip Being Rolled = .063"

% Reduction	P_{max} Ton-in/rev.		Design Vector X_1 Inch.	X_2 Ton/Sq.in	X_3 Ton/Sq.in	Obj.fn. (deflection) 10^{-4} in.	Power used Ton-in/rev.	No. of Iterations	Computer Time Min:Sec.
10%	800.00	Initial	3.000	0.794	0.882	32.590	-	12	05 : 05
		Optimal	9.442	2.464	2.900	2.804	785.64		
20%	800.00	Initial	3.000	0.794	0.992	42.160	-	65	11 : 56
		Optimal	7.723	2.082	2.416	3.580	769.28		
30%	800.00	Initial	3.000	0.794	1.133	51.650	-	52	12 : 15
		Optimal	6.895	1.055	1.340	5.544	783.472		
40%	800.00	Initial	3.000	0.794	1.322	57.03	-	51	12 : 06
		Optimal	6.117	0.752	1.170	8.280	786.4		
10%	900.00	Initial	9.442	2.464	2.900	2.804	-	21	05 : 03
		Optimal	10.242	2.320	1.613	1.476	880.84		
20%	900.00	Initial	7.723	2.082	2.416	3.580	-	20	05 : 10
		Optimal	8.371	2.257	2.638	3.019	874.69		
30%	900.00	Initial	6.895	1.055	1.340	5.544	-	28	10 : 09
		Optimal	7.430	1.039	1.714	4.700	880.59		
40%	900.00	Initial	6.895	0.752	1.170	8.280	-	20	06 : 50
		Optimal	6.632	0.770	1.228	6.871	885.90		
10%	1000.00	Initial	9.442	2.464	2.900	2.804	-	41	12 : 07
		Optimal	11.560	2.373	2.560	1.185	976.31		
20%	1000.00	Initial	7.723	2.082	2.416	3.580	-	62	12 : 07
		Optimal	9.156	1.742	2.103	2.524	961.47		
30%	1000.00	Initial	6.895	1.055	1.340	5.544	-	41	13 : 56
		Optimal	8.045	0.998	1.378	3.972	980.64		
40%	1000.00	Initial	6.117	0.752	1.170	8.280	-	33	14 : 07
		Optimal	7.102	0.827	1.382	5.903	983.93		

Initial Thickness of Strip Being Rolled = .013"

S.No.	% Reduction	P_{max} Ton-in/rev.		Design Vector X_1 in.	X_2 Ton/Sq.in.	X_3 Ton/Sq.in.	Obj.fn. (deflection) 10^{-4} in.	No. of Iterations.	Computer time
1	10%	800.00	Initial	5.000	3.846	4.273	5.485	18	20 : 27
			Optimal	9.090	11.153	21.356	2.231		
2	20%	800.00	Initial	5.000	3.846	4.808	7.726	12	13 : 39
			Optimal	8.106	11.38	14.0	3.892		
3	30%	800.00	Initial	4.000	3.846	5.495	16.28	6	06 : 50
			Optimal	7.122	12.054	9.000	6.306		
4	40%	800.00	Initial	4.000	3.846	6.410	19.900	13	13 : 20
			Optimal	6.190	15.714	20.218	9.930		
5	10%	900.00	Initial	5.000	3.846	4.273	5.485	27	21 : 03
			Optimal	9.238	23.993	20.84	2.208		
6	20%	900.00	Initial	5.000	3.846	4.808	7.726	14	13 : 53
			Optimal	8.108	22.28	17.34	3.876		
7	30%	900.00	Initial	4.000	3.846	5.495	16.280	13	13 : 23
			Optimal	7.011	24.875	25.615	6.268		
8	40%	900.00	Initial	4.000	3.846	6.410	19.900	14	13 : 50
			Optimal	6.265	25.095	27.910	9.820		
9	10%	1000.00	Initial	5.000	3.846	4.273	5.485	24	21 : 08
			Optimal	9.204	30.00	30.62	2.193		
10	20%	1000.00	Initial	5.000	3.846	4.808	7.726	17	12 : 58
			Optimal	8.109	30.200	28.000	3.846		
11	30%	1000.00	Initial	4.000	3.846	5.495	16.280	20	14 : 56
			Optimal	7.161	30.35	31.0	6.195		
12	40%	1000.00	Initial	4.000	3.846	6.410	19.900	18	13 : 11
			Optimal	6.276	32.000	41.8	9.747		

TABLE 7

Data : Initial thickness of the strip being rolled = 0.063"

Maximum allowable deflection = 5×10^{-4} inches

S.No.	% Reduction		Design Vector x_1 in.	x_2 Ton/sq in.	x_3 Ton/sq. in	Obj.fn. (Power Required) Ton-in/rev.	Deflection of Rolls 10^{-4} in	No. of iterations	Computer time Min : Sec.
1	10%	Initial	7.441	7.464	6.95	790.32	2.804	12	12 : 27
		Optimal	7.262	1.2×10^{-3}	4.8×10^{-3}	374.24	3.005		
2	20%	Initial	7.723	2.082	2.416	774.17	3.580	3	03 : 57
		Optimal	7.681	2.96×10^{-3}	4.44×10^{-3}	646.88	3.640		
3	30%	Initial	8.045	0.998	1.377	980.64	3.973	7	08 : 14
		Optimal	8.013	2.57×10^{-3}	2.88×10^{-3}	918.18	4.015		
4	40%	Initial	8.500	0.794	1.321	1768.81	4.13	13	14 : 29
		Optimal	8.183	4.2×10^{-3}	1.06×10^{-2}	1168.73	4.397		

CHAPTER 4

CONCLUSIONS AND RECOMMENDATIONS

4.1 CONCLUSIONS

4.1.1 Analysis of Cold Rolling Process

The present analysis gives reasonable results as far as roll forces and torques are concerned. These results are expected to come very close to the experimental or actual values if the modified roll radius, after considering the elasticity of rolls, is used in the analysis.

The pressure distribution along the work-roll interface agrees well with the general shape observed during experimental investigations [25].

The deformation pattern and the flow of material gives a fair idea about the elastic and plastic deformation taking place in the strip.

4.1.2 Design of Cold Rolling Process

The following conclusions can be drawn from the optimum design study of single pass rolling:

(i) In thinner strips, the tensions are more effective in improving the quality of the product

while in the case of thicker strips, an increase in the roll radius gives a better quality of the product.

(ii) In the case of thinner strips, when the total power available is the limiting factor, then the power supplied through tensions is a better way to improve the quality whereas in the case of thicker strips, the power supplied should be through the torque applied across the rolls for improving the quality.

(iii) In order to reduce the running costs, i.e, to reduce the power, the tensions have to be made zero.

In the case of multipass cold rolling, the best way to improve the quality of the product is to have same dimensions for the rolls of all mills, almost equal tensions at all the places and to give the maximum possible reduction in the first pass itself. This conclusion is based on the results of one example and is to be supported with further numerical results.

4.2 RECOMMENDATION FOR FUTURE WORK

The finite element analysis can be improved by taking higher order terms in the displacement model. Besides, the deflection of the rolls should be taken into account to obtain better results. This analysis can be extended to rectangular bars and rods, where spread is considerable, by considering a 3-dimensional element.

The finite element analysis can also be used for thermal analysis to find the temperature distribution and hence the material properties of each element. This information, in turn, can be used to make an elasto-plastic analysis of a hot rolling process.

In the case of design problem, the other types of objective functions like shape function, rate of production etc. can also be considered. Besides, by considering the change in friction due to change in speed of rolls, velocity can be taken as a design variable.

Similarly, the rate of production can be taken as the objective function in the case of tandem mill. Also the problem can be solved for finding the optimum design of tandem mills having more than two single pass rolling mills in series. In the present study, rolling mills having two rolls only are considered. The problem can be extended for the design of rolling mills having back up rolls, which are quite extensively used in

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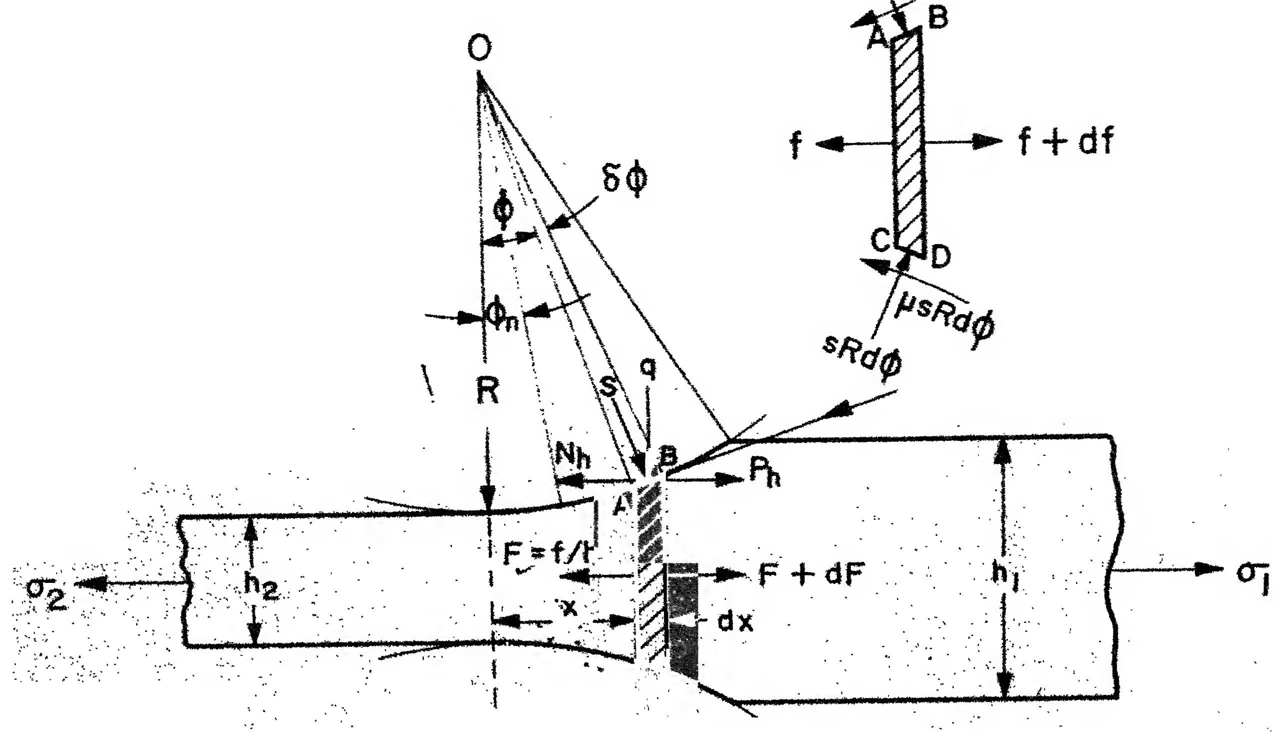


Fig. I Element In Roll Gap

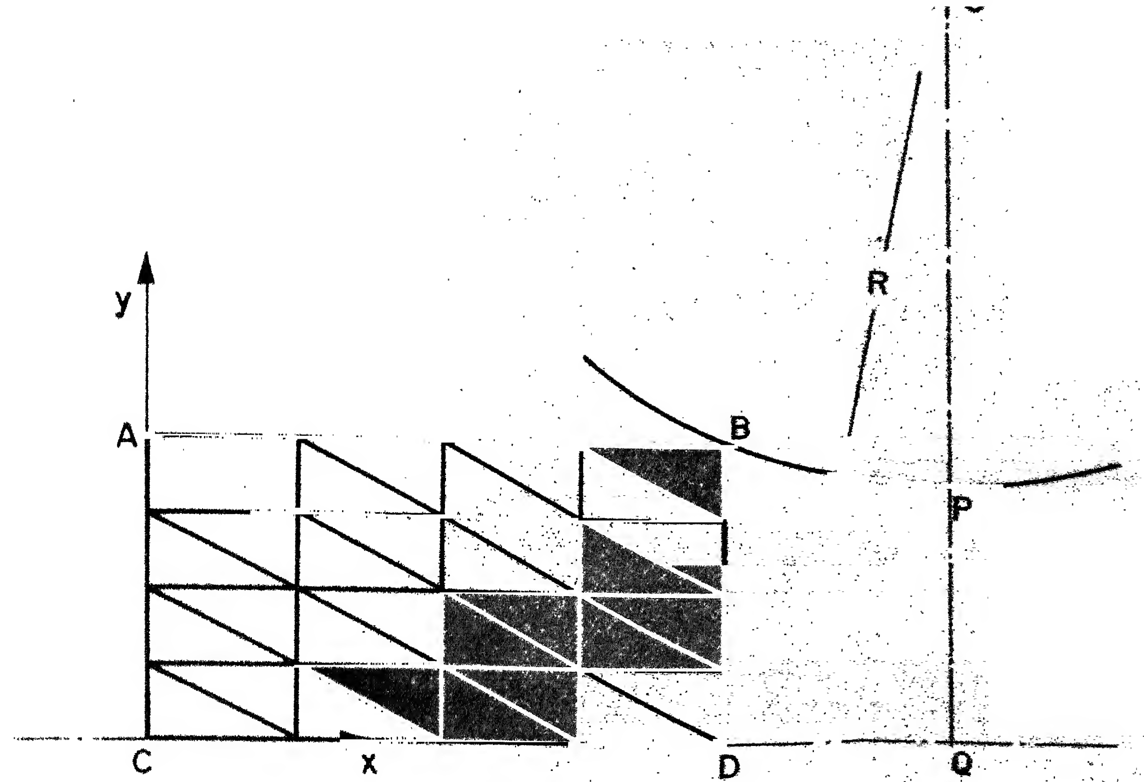


Fig.(3A) Strip To Be Rolled Divided Into Triangular Finite Element

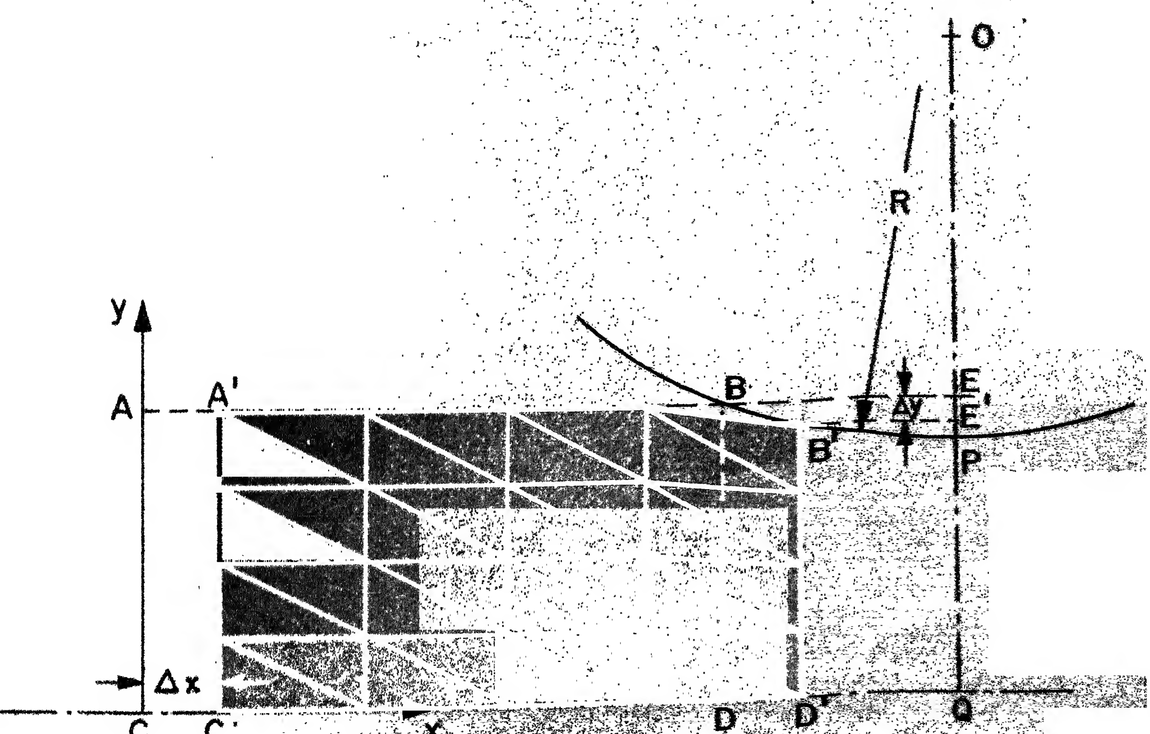


Fig.(3B) Strip Of Material After Displacement Δx

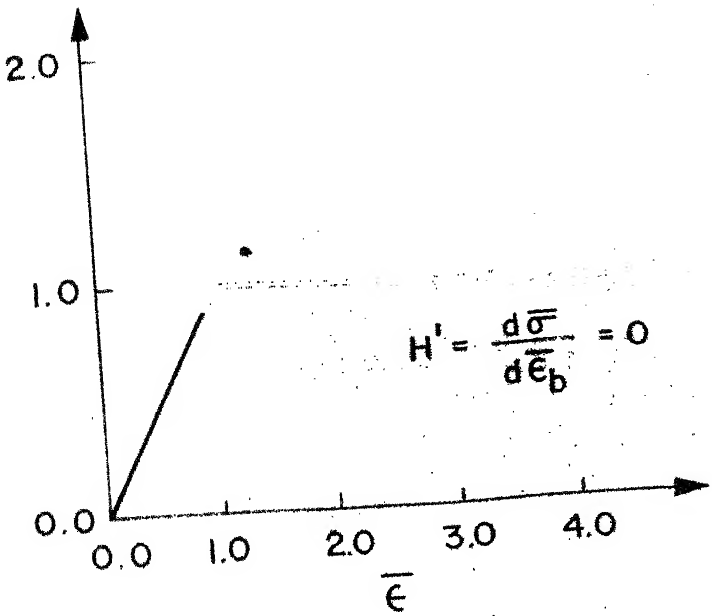


Fig. (4A) Elastic–Perfectly Plastic Type Stress Strain Relation.

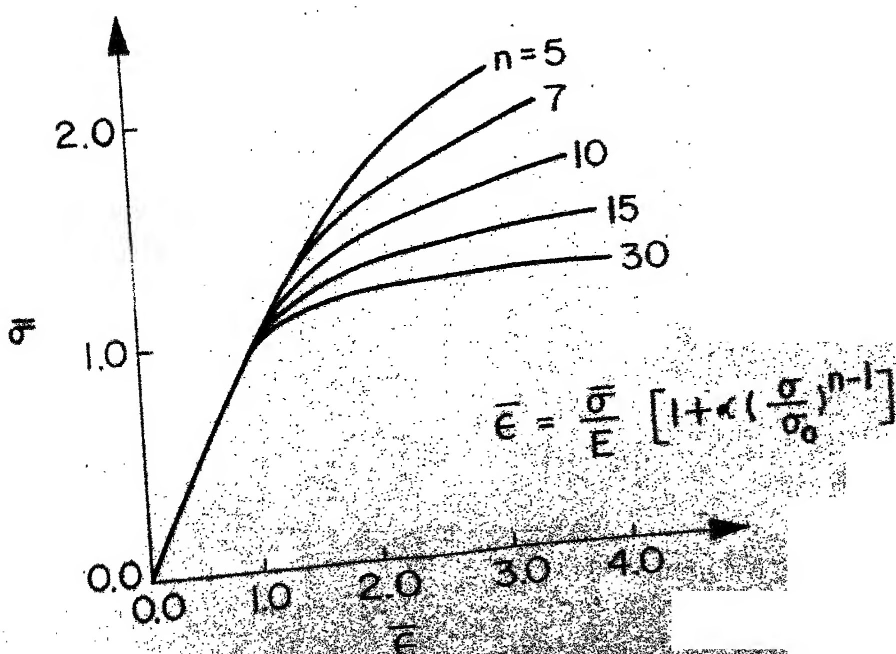


Fig.(4B) Ramberg—Osgood Type Stress Strain Relation.

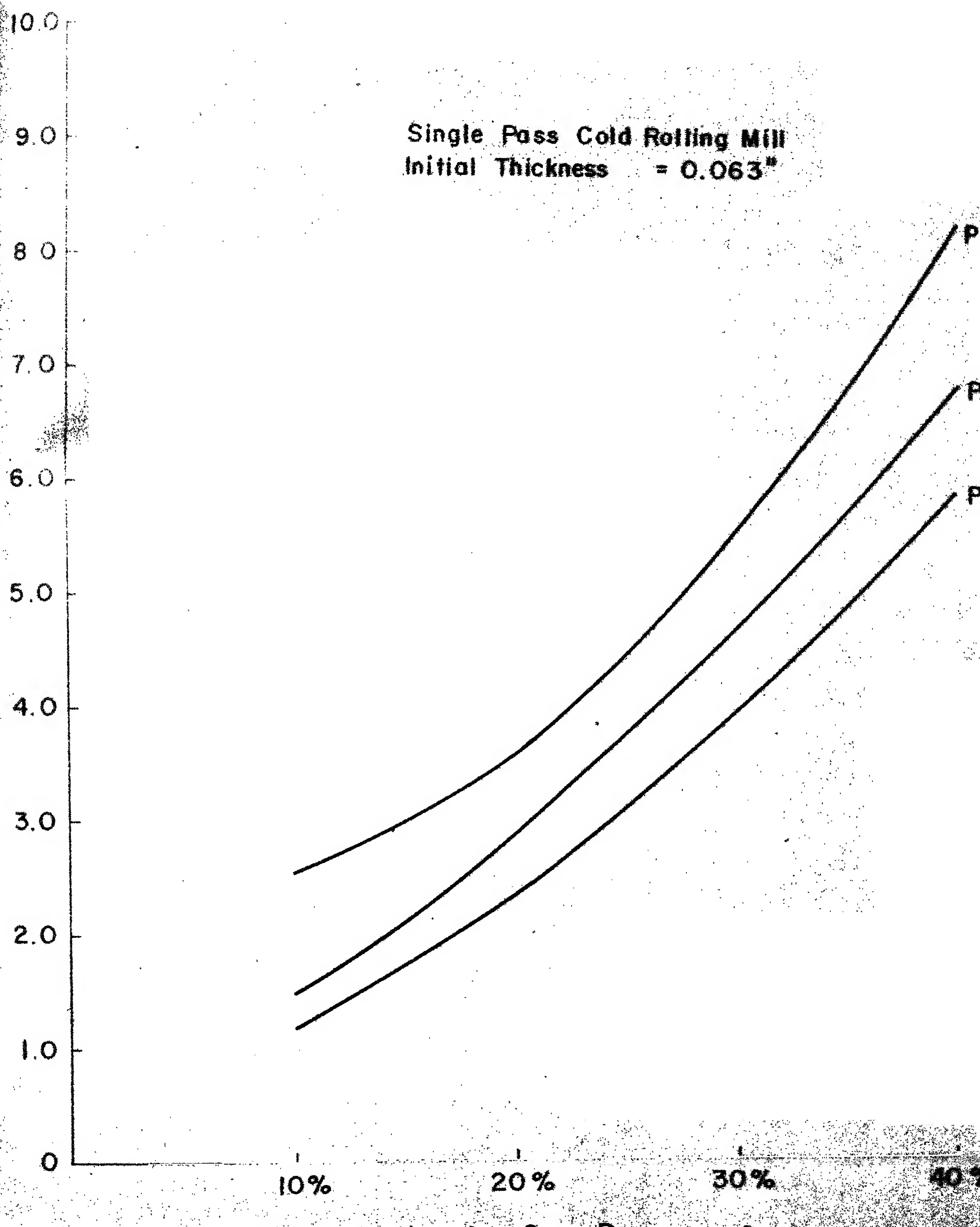


Fig. (12) Variation Of Roll Deflection With % Reductio
And Max Power Available

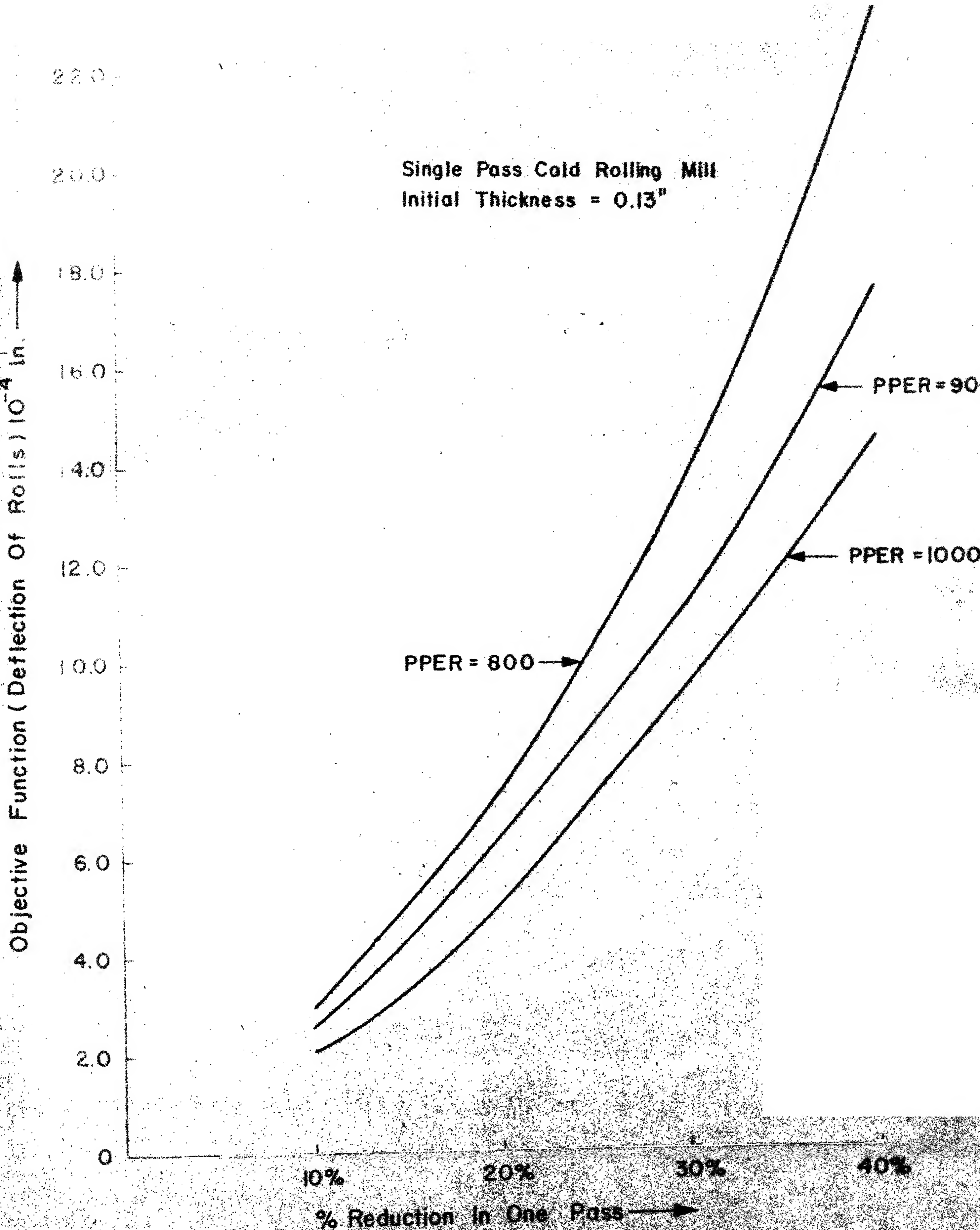


Fig. (13) Variation Of Roll Deflection With % Reduction And Max. Power Available

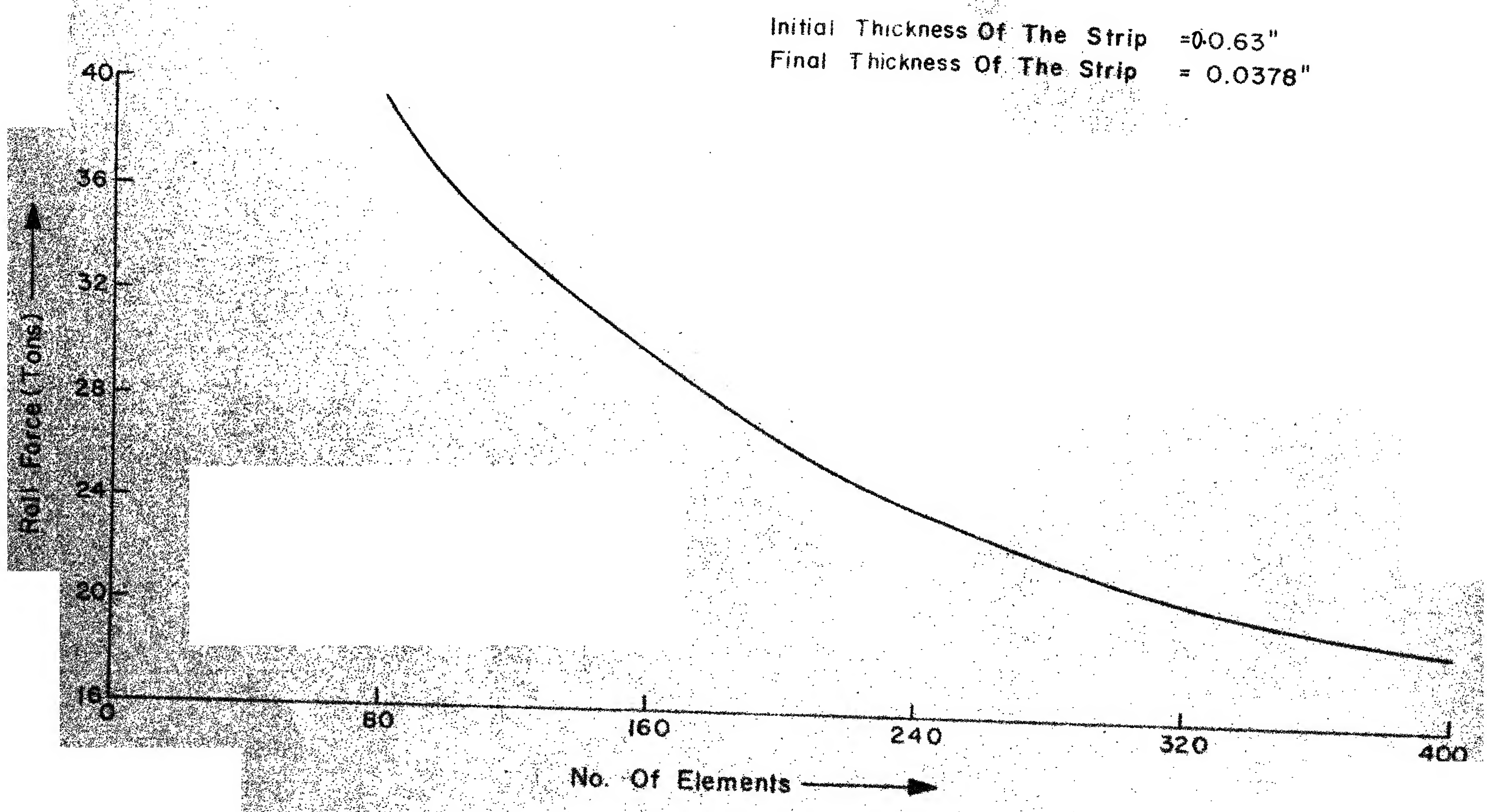


Fig. 14. Convergence Of Finite Element Method

APPENDIX A

VERTICAL DISPLACEMENT (ΔY) OF A NODE IN CONTACT WITH THE ROLLS FOR DISPLACEMENT (ΔX) IN X DIRECTION

Let the coordinates of any node i be (x_i, y_i), and the radius of rolls be R. Let the input and the output thickness of the strip or the bar rolled be H_1 and H_2 respectively.

From Fig. (3B)

$$CQ = CD + DQ$$

$$= CD + \sqrt{R^2 - OE^2}$$

$$OE = R - \frac{h_1 - h_2}{2}$$

Hence

$$CQ = CD + \sqrt{R(h_1 - h_2) - (h_1 - h_2)^2/4} \qquad \text{(A-1)}$$

The new coordinates after a movement of x are

$(x_i + \Delta x)$ and $(y_i + \Delta y)$

Therefore

$$E'P = OP - OE'$$

$$= R - OE'$$

$$OE' = \sqrt{R^2 - B'E'^2}$$

$$B'E' = D'Q = CQ - (x_i + \Delta x)$$

From Eq. (A-1), one gets

$$B'E' = CD + \sqrt{R(h_1 - h_2) - (h_1 - h_2)^2 / 4} - (x_i + \Delta x) \qquad \text{(A-2)}$$

$$\begin{aligned} y_i + \Delta y &= E'P + PQ \\ &= \frac{h_2}{2} + R - \sqrt{R^2 - B'E'^2} \end{aligned}$$

$$\therefore \Delta y = v = \frac{h_2}{2} + R - \sqrt{R^2 - B'E'^2} - y_i \qquad \text{(A-3)}$$

$B'E'$ in Eq. (A-3) is defined in Eq. (A-2).

This can be implemented on computer to calculate Δy for a unit incremental displacement Δx.

APPENDIX B

DERIVATION OF FUNCTION F_3 AND F_5

Inserting the values of s^+ and s^- into Eq. (3.7) gives

$$P = R'\bar{k}\left[\int_0^{\phi_n} \frac{h}{h_2}\left(1 - \frac{\sigma_2}{\bar{k}}\right) e^{\mu H}\, d\phi + \int_{\phi_n}^{\phi_1} \frac{h}{h_1}\left(1 - \frac{\sigma_1}{\bar{k}}\right) e^{\mu (H_1 - H)}\, d\phi\right.$$

Assume

$$a = \mu\sqrt{\frac{R'}{h_2}}$$

$$b = \frac{1 - \sigma_2 / \bar{k}}{1 - \sigma_1 / \bar{k}}$$

$$r = \frac{1 - h_2}{h_1}$$

For ease of calculation put $x = \sqrt{\frac{R'}{h_2}}\ \phi$

Hence $dx = \sqrt{\frac{R'}{h_2}}\ d\phi$

One gets from Fig. (1), the thickness of transverse slice at distance x,

$$\frac{h}{h_2} = \frac{h_2 + R'\phi^2}{h_2} = 1 + x^2$$

from this one gets

$$\frac{h}{h_1} = \frac{h_2}{h_1} \cdot \frac{h}{h_2} = (1 - r)(1 + x^2)$$

$$H = 2\, a \tan^{-1} x \text{ and } H_1 = 2\, a \tan^{-1} x_1$$

$$\text{Now } P = R\, \bar{k}\, (1 - \frac{\sigma_1}{\bar{k}}) \Big[\int_0^{x_n} b\, (1 + x^2)\, e^{2\, a \tan^{-1} x} \cdot \sqrt{\frac{h_2}{R}}\, dx + \int_{x_n}^{x_1} (1 - r)\, (1 + x^2)\, e^{2\, a \tan^{-1} x} \cdot \sqrt{\frac{h_2}{R}}\, d\, x \Big]$$

$$\text{or } P = \bar{k}\, (1 - \frac{\sigma_1}{\bar{k}}) \sqrt{R\, (h_1 - h_2)} \Big[\sqrt{\frac{1 - r}{r}} \cdot \int_0^{x_n} b\, (1 + x^2)\, e^{2\, a \tan^{-1} x}\, dx + (1 - r) \sqrt{\frac{1 - r}{r}}\, e^{2\, a \tan^{-1} x_1} \cdot \int_{x_n}^{x_1} (1 + x^2)\, e^{-2\, a \tan^{-1} x}\, dx \Big]$$

Comparing above equation with Eq. (3.11) one gets

$$f_3\, (a,\, r,\, b) = b \sqrt{\frac{1-r}{r}} \int_0^{x_n} b\, (1 + x^2)\, e^{2a \tan^{-1} x}\, dx + (1-r) \sqrt{\frac{1-r}{r}}\, e^{2a \tan^{-1} x_1} \int_{x_n}^{x_1} (1 + x^2)\, e^{-2\, a \tan^{-1} x} \cdot dx$$

Roll torque function f_5 can be derived by inserting the value of s^+ and s^- into Eq. (3.10)

$$G = R R' \bar{k} \left(1 - \frac{\sigma_1}{\bar{k}}\right)\left[\int_0^{\phi_n} \frac{h}{h_2} \, b \, e^{\mu H} \phi \, d\phi + \int_{\phi_n}^{\phi} \frac{h}{h_1} e^{\mu (H_1 - H)} \phi \, d\phi \right]$$

for ease of calculation a new function x given by $\sqrt{\frac{R'}{h_2}}\,\phi$ is used. Writing $\frac{h}{h_1}$, μH and μH_1 in terms of it and substituting in above equation one gets

$$G = R \bar{k} \left(1 - \frac{\sigma_1}{\bar{k}}\right)(h_1 - h_2)\left[\frac{h_2}{(h_1 - h_2)} \left\{ b \int_0^{x_n} (1 + x^2) \, . \, x \, e^{2 a \tan^{-1} x} dx + (1 - r) \, e^{2a \tan^{-1} x_1} \, . \int_{x_n}^{x_1} (1 + x^2) \, x \, e^{-2 a \tan^{-1} x} dx \right\} \right]$$

Comparing above equation for G with Eq. (3.12) one gets

$$f_5 = (a, r, b) = \frac{h_2}{h_1 - h_2}\left[b \int_0^{x_n} (1 + x^2) \, x \, e^{2 a \tan^{-1} x} dx + (1 - r) \, e^{2 a \tan^{-1} x_1} \int_{x_n}^{x_1} (1 + x^2) \, . \, e^{-2 a \tan^{-1} x} dx \right]$$

These values for f_3 and f_5 can be calculated by numerical methods for any values of non dimensional constants a, r and b.

```
      NM=0
      DELX1=DELX
   10 CONTINUE
      PRINT2,NI,MM
      CALL INCDIS
C
C                                  FORM STIFFNESS MATRIX
C
      CALL FORMK
C
C                                  SOLVE THE EQUATIONS
C
      CALL SOLVE
C
C                                  CALCULATE THE STRESSES IN EACH ELEMENT
C
      CALL STRESS
      CALL SIGBAR(ANETA,MM,NI,NII)
      IF(MM.EQ.1)GOTO20
      DELX=DELX1*(ANETA-1.0)
      GOTO102
   20 CONTINUE
      IF(DELX.GT.0.01)GOTO101
      GOTO102
  101 DELX=0.01
  102 CONTINUE
      NI=NI+1
      IF(NI.GT.NPRE)GOTO50
      GOTO10
   50 CONTINUE
    1 FORMAT(9I5)
    2 FORMAT(22H NUMBER OF ITERATIONS=,I5,20X,3HMM=,I2)
      STOP
      END
      SUBROUTINE GDATA
      COMMON/CONTR/TITLE(12),NP,NE,NDF,NCN,NLD,NMAT,NSZF,NPRE,LMCN
      COMMON/CONTS/CORD(300,2),NOP(472,3),ORT(1,2)
      COMMON/CONTT/IMP(600),X(600),B(600)
      COMMON/CONTU/DELX,RAD,H1,H2,AD,Y
      COMMON/CONTY/NH,NV
C
C                                  READ AND PRINT TITLE AND CONTROL
C
      READ7,TITLE
      PRINT100,TITLE
      READ1,NP,NE,NLD,NDF,NMAT,NPRE
      PRINT1,NP,NE,NLD,NDF,NMAT,NPRE
C
C                                  READ AND PRINT MATERIAL DA3A
C
```

```
      READ8,(N,(ORT(N,I),I=1,2),L=1,NMAT)
      PRINT108
      PRINT8,(N,(CRT(N,I),I=1,2),N=1,NMAT)
C
C                                  GENERATE NODAL POINT DATA
C
      READ1,NV,NH
      READ6,STEPH,STEPV
      A2=NV-1
      DO60I=1,NV
      A1=0.0
      DO50J=I,NP,NV
      CORD(J,1)=A1*STEPH
      CORD(J,2)=A2*STEPV
      A1=A1+1.0
   50 CONTINUE
      A2=A2-1.0
   60 CONTINUE
C
C                                  GENERATE ELEMENT DATA
C
      READ1,II1,II2,II3,JJ1,JJ2,JJ3,I1,I2
      NHH=NH-1
      NVV=NV-1
      DO80I=1,NHH
      IJ1=II1
      IJ2=II2
      IJ3=II3
      JI1=JJ1
      JI2=JJ2
      JI3=JJ3
      DO70J=1,NVV
      NOP(I1,1)=IJ1
      NOP(I1,2)=IJ2
      NOP(I1,3)=IJ3
      NOP(I2,1)=JI1
      NOP(I2,2)=JI2
      NOP(I2,3)=JI3
      I1=I1+1
      I2=I2+1
      IJ1=IJ1+1
      IJ2=IJ2+1
      IJ3=IJ3+1
      JI1=JI1+1
      JI2=JI2+1
      JI3=JI3+1
   70 CONTINUE
      I1=I1+NVV
      I2=I2+NVV
      II1=II1+NV
```

```
      II2=II2+NV
      II3=II3+NV
      JJ1=JJ1+NV
      JJ2=JJ2+NV
      JJ3=JJ3+NV
   50 CONTINUE
      NHV=NHH*NVV*2
C
C                                      READ AND PRINT BOUNDRY DATA
C
      PRINT104
      NSZF=NP*NDF
      READ4,(IMP(I),I=1,NSZF)
      READ10,DELX,RAD,H1,H2,AD
      READ11,Y
      PRINT10,DELX,RAD,H1,H2,AD
      PRINT11,Y
C
C                                      PRINT INPUT DATA
C
      PRINT102
      PRINT2,(N,(CORD(N,M),M=1,2),N=1,NP)
      PRINT103
      PRINT3,(N,(NOP(N,M),M=1,3),N=1,NE)
  200 CONTINUE
    1 FORMAT(9I5)
    2 FORMAT(3(I5,2F10.6))
    3 FORMAT(26I5)
    4 FORMAT(30I1)
    5 FORMAT(F8.3)
    6 FORMAT(2F12.8)
    7 FORMAT(12A6)
    8 FORMAT(I5,2E10.3)
    9 FORMAT(I5,E15.8)
   10 FORMAT(5F8.4)
   11 FORMAT(E10.3)
  100 FORMAT(1H ,12A6)
  102 FORMAT(12H NODAL POINTS)
  103 FORMAT(9H ELEMENTS)
  104 FORMAT(20H BOUNDARY CONDITIONS)
  105 FORMAT(/30H  GIVEN COMPONENTS OF VECTOR X)
  106 FORMAT(/30H  GIVEN COMPONENTS OF VECTOR B)
  108 FORMAT(20H MATERIAL PROPERTIES)
      RETURN
      END
      SUBROUTINE INCDIS
      COMMON/CONTS/CORD(300,2),NOP(472,3),ORT(1,2)
      COMMON/CONTR/TITLE(12),NP,NE,NDF,NCN,NLD,NMAT,NSZF,NPRE,LMCN
      COMMON/CONTT/IMP(600),X(600),B(600)
      COMMON/CONTU/DELX,RAD,H1,H2,AD,Y
```

```
      COMMON/CONTY/NH,NV
      DO15I=1,NSZF
      IF(IMP(I).EQ.0)GOTO10
      B(I)=0.0
      GOTO15
   10 B(I)=0.0
   15 CONTINUE
      DO20I=1,NP
   20 CORD(I,1)=CORD(I,1)+DELX
      DO30I=1,NP,NV
      IF(CORD(I,1).LE.AD)GOTO30
      X11=SQRT(RAD*(H1-H2)-(H1-H2)**2/4.0)
      X1=AD+ABS(X11)
      IF(CORD(I,1).GT.X1)GOTO25
      Y1=RAD+H2/2.0
      AP=SQRT(RAD**2-(CORD(I,1)-X1)**2)
      YN=Y1-ABS(AP)
      B(2*I)=YN-CORD(I,2)
      IMP(2*I)=1
      B(2*I-1)=0.0
      IMP(2*I-1)=1
      PRINT60,I
      GOTO30
   25 CONTINUE
      B(2*I)=0.0
      IMP(2*I)=0
      B(2*I-1)=0.0
      IMP(2*I-1)=0
      PRINT40,I
   30 CONTINUE
   40 FORMAT(I5,*TH NODE HAS COMEOUT OF THE ROLLS*)
   50 FORMAT(*    IMP&S=*,100I1)
   60 FORMAT(I5,*TH NODE IS BETWEEN THE ROLLS*)
      RETURN
      END
      SUBROUTINE FORMK
      COMMON/CONTR/TITLE(12),NP,NE,NDF,NCN,NLD,NMAT,NSZF,NPRE,LMCN
      COMMON/CONTS/CORD(300,2),NOP(472,3),ORT(1,2)
      COMMON/ABG/SK(510,14)
      COMMON/ABC/ESTIFM(6,6)
      COMMON/BCD/MB,NNN
      DIMENSION KK(6)
      DO300N=1,NSZF
      DO300M=1,MB
  300 SK(N,M)=0.0
      MBI=0
      DO500N=1,NE
      CALL STIFT2(N)
      KK(2)=2*NOP(N,1)
      KK(4)=2*NOP(N,2)
```

```
      KK(6)=2*NOP(N,3)
      KK(1)=KK(2)-1
      KK(3)=KK(4)-1
      KK(5)=KK(6)-1
      DO400I=1,6
      II=KK(I)
      DO400J=1,6
      IF(KK(J).LT.II)GOTO400
      JJ=KK(J)-II+1
      SK(II,JJ)=SK(II,JJ)+ESTIFM(I,J)
      IF(MBI.GT.JJ)GOTO400
      MBI=JJ
  400 CONTINUE
  500 CONTINUE
      MB=MBI
      PRINT600,MBI
  600 FORMAT(*  BANDWIDTH=*,I5)
      RETURN
      END
      SUBROUTINE STIFT2(N)
      COMMON/ABC/ESTIFM(6,6)
      COMMON/ABD/A(3,6),B(3,6)
      COMMON/ABE/AREA
      CALL STBAM(N)
C
C                                   ESTIFM IS STIFFNESS MATRIX
C
      DO210I=1,6
      DO210J=1,6
      ESTIFM(I,J)=0.0
      DO210K=1,3
  210 ESTIFM(I,J)=ESTIFM(I,J)+B(K,I)*A(K,J)*AREA
      RETURN
      END
      SUBROUTINE STBAM(N)
      COMMON/CONTR/TITLE(12),NP,NE,NDF,NCN,NLD,NMAT,NSZF,NPRE,LMCN
      COMMON/CONTS/CORD(300,2),NOP(472,3),ORT(1,2)
      COMMON/CONTU/DELX,RAD,H1,H2,AD,Y
      COMMON/ABD/A(3,6),B(3,6)
      COMMON/ABE/AREA
      COMMON/ABF/FORCE(410,4),STRAN(410,3),SIGB(410),INP(410),EFST(410)
      DIMENSION ESTIFM(6,6)
C
C                                   DETERMINE ELEMENT CONNECTION
C
      I=NOP(N,1)
      J=NOP(N,2)
      K=NOP(N,3)
      L=1
C
```

```
C                                         SET UP LOCAL COORDINATE SYSTEM
C
      AJ=CORD(J,1)-CORD(I,1)
      AK=CORD(K,1)-CORD(I,1)
      BJ=CORD(J,2)-CORD(I,2)
      BK=CORD(K,2)-CORD(I,2)
      AREA=(AJ*BK-AK*BJ)/2.0
      AREA=ABS(AREA)
      IF(AREA.LE.0.0)GOTO220
C
C                                         FORM STRAIN DISPLACEMENT MATRIX
C
      A(1,1)=BJ-BK
      A(1,2)=0.0
      A(1,3)=BK
      A(1,4)=0.0
      A(1,5)=-BJ
      A(1,6)=0.0
      A(2,1)=0.0
      A(2,2)=AK-AJ
      A(2,3)=0.0
      A(2,4)=-AK
      A(2,5)=0.0
      A(2,6)=AJ
      A(3,1)=AK-AJ
      A(3,2)=BJ-BK
      A(3,3)=-AK
      A(3,4)=BK
      A(3,5)=AJ
      A(3,6)=-BJ
      DO201I=1,3
      DO201J=1,6
  201 A(I,J)=A(I,J)/(2.0*AREA)
C
C                                         FORM STRESS-SRRAIN MATRIX
C
      IF(INP(N).EQ.1)GOTO202
      COMM=ORT(L,1)/((1.0+ORT(L,2))*(1.0-ORT(L,2)*2.0))
      STIFM(1,1)=COMM*(1.0-ORT(L,2))
      STIFM(1,2)=COMM*ORT(L,2)
      STIFM(1,3)=0.0
      STIFM(2,1)=STIFM(1,2)
      STIFM(2,2)=STIFM(1,1)
      STIFM(2,3)=0.0
      STIFM(3,1)=0.0
      STIFM(3,2)=0.0
      STIFM(3,3)=ORT(L,1)/(2.0*(1.0+ORT(L,2)))
      GOTO204
  202 COM=ORT(L,1)/(1.0+ORT(L,2))
      COM1=(1.0-ORT(L,2))/(1.0-2.0*ORT(L,2))
```

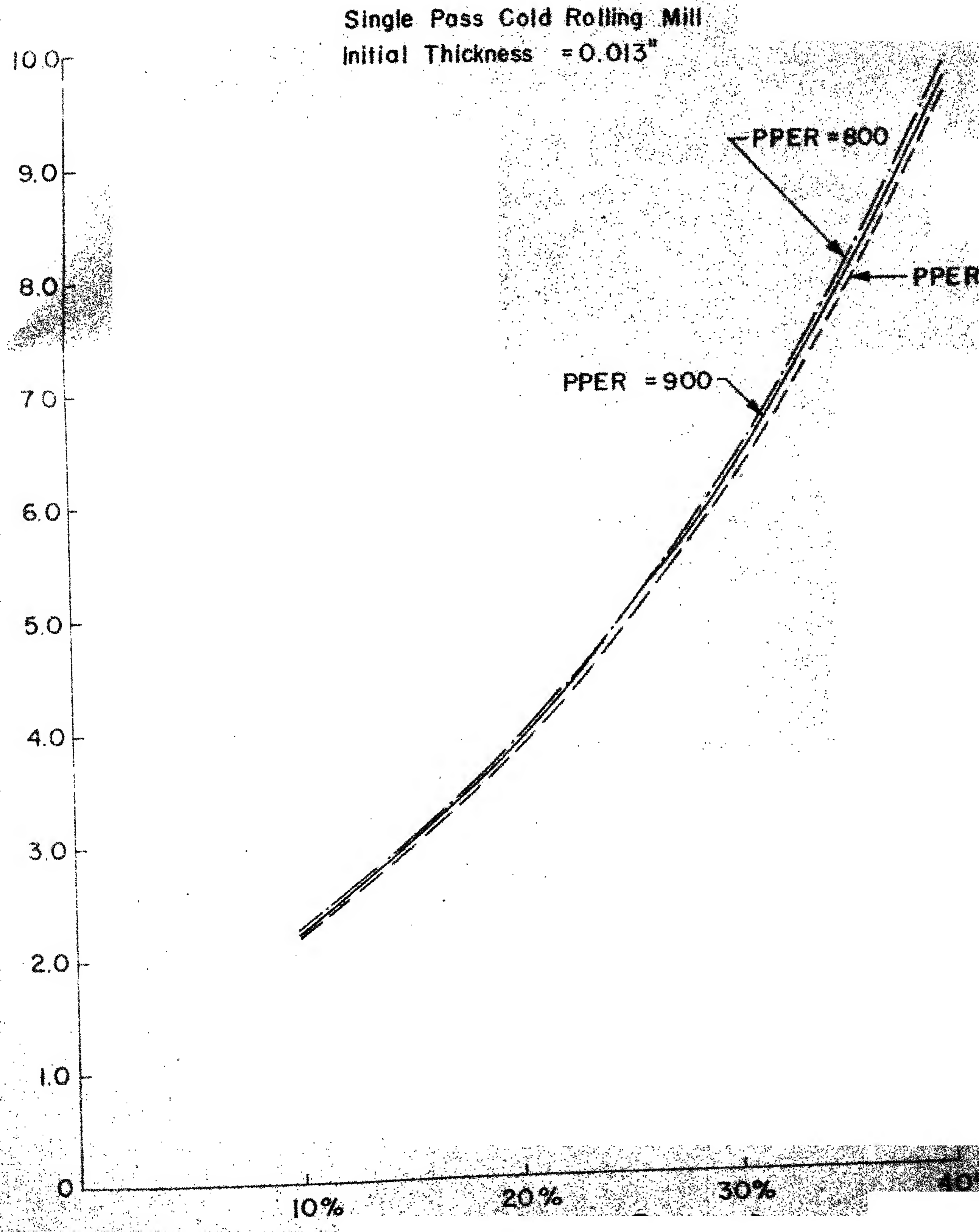
```
      COM2=ORT(L,2)/(1.0-2.0*ORT(L,2))
      HP=0.0
      COMS=2.0*(1.0+2.0*(1.0+ORT(L,2))*HP/(ORT(L,1)*3.0))*SIGB(N)**2/3.
      DJ=FORCE(N,1)+FORCE(N,2)+FORCE(N,4)
      DSIGX=FORCE(N,1)-DJ/3.0
      DSIGY=FORCE(N,2)-DJ/3.0
      ESTIFM(1,1)=COM*(COM1-DSIGX**2/COMS)
      ESTIFM(1,2)=COM*(COM2-DSIGX*DSIGY/COMS)
      ESTIFM(1,3)=COM*(-DSIGX*FORCE(N,3)/COMS)
      ESTIFM(2,1)=ESTIFM(1,2)
      ESTIFM(2,2)=COM*(COM1-DSIGY**2/COMS)
      ESTIFM(2,3)=COM*(-DSIGY*FORCE(N,3)/COMS)
      ESTIFM(3,1)=ESTIFM(1,3)
      ESTIFM(3,2)=ESTIFM(2,3)
      ESTIFM(3,3)=COM*(0.5-FORCE(N,3)**2/COMS)
  204 CONTINUE
      DO205I=1,3
      DO205J=1,6
      B(I,J)=0.0
      DO205K=1,3
  205 B(I,J)=B(I,J)+ESTIFM(I,K)*A(K,J)
      RETURN
C
C                                 ERROR EXIT FOR BAD CONNECTION
C
  220 PRINT100,N
  100 FORMAT(33H ZERO OR NEGATIVE AREA ELEMENT NO,14/21H EXECUTION TERMI
     1NATED)
      STOP
      END
      SUBROUTINE SOLVE
      DIMENSION XX(600),III(600)
      COMMON/CONTR/TITLE(12),NP,NE,NDF,NCN,NLD,NMAT,NSZF,NPRE,LMCN
      COMMON/CONTT/IMP(600),X(600),B(600)
      COMMON/ABG/SK(510,14)
      COMMON/BCD/MB,NNN
      COMMON/XYZ/BB(600)
      DO60N=1,NSZF
      IF(IMP(N).EQ.1)GOTO60
      DO40L=1,MB
      IF(SK(N,L).EQ.0.0)GOTO40
      I=N+L-1
      IF(IMP(I).EQ.0)GOTO40
      B(N)=B(N)-SK(N,L)*X(I)
   40 CONTINUE
      DO50L=2,MB
      I=N+1-L
      IF(I.LE.0)GOTO50
      IF(IMP(I).EQ.0)GOTO50
      B(N)=B(N)-SK(I,L)*X(I)
```

```
   0 CONTINUE
  60 CONTINUE
C     FORWARD REDUCTION OF MATRIX (GAUSS ELIMINATION)
      DO790N=1,NSZF
      IF(IMP(N).EQ.1)GOTO790
      DO780L=2,MB
      IF(SK(N,L).EQ.0.0)GOTO780
      I=N+L-1
      IF(IMP(I).EQ.1)GOTO780
      C=SK(N,L)/SK(N,1)
      J=0
      DO750K=L,MB
      J=J+1
      II=I+J-1
      IF(IMP(II).EQ.1)GOTO750
      SK(I,J)=SK(I,J)-C*SK(N,K)
 750  CONTINUE
      SK(N,L)=C
 780  CONTINUE
 790  CONTINUE
C     FORWARD REDUCTION OF CONSTANTS (GAUSS ELIMINATION)
      DO830N=1,NSZF
      IF(IMP(N).EQ.1)GOTO830
      DO820L=2,MB
      IF(SK(N,L).EQ.0.0)GOTO820
      I=N+L-1
      IF(IMP(I).EQ.1)GOTO820
      B(I)=B(I)-SK(N,L)*B(N)
 820  CONTINUE
      B(N)=B(N)/SK(N,1)
 830  CONTINUE
C     SOLVE FOR UNKNOWNS BY BACK SUBSTITUTION .
      DO860M=2,NSZF
      N=NSZF+1-M
      IF(IMP(N).EQ.1)GOTO860
      DO850L=2,MB
      IF(SK(N,L).EQ.0.0)GOTO850
      K=N+L-1
      IF(IMP(K).EQ.1)GOTO850
      B(N)=B(N)-SK(N,L)*B(K)
 850  CONTINUE
 860  CONTINUE
  15  FORMAT(/35H  COMPUTED COMPONENTS OF VECTOR X)
      JJ=0
      DO14I=1,NSZF
      IF(IMP(I).EQ.1)GOTO14
      X(I)=B(I)
      JJ=JJ+1
      XX(JJ)=X(I)
      III(JJ)=I
```

```
      FST(I)=FFST(I)+YZX
 100  CONTINUE
      DO140I=1,NE
      FORCE(I,4)=FORCE(I,4)+FINT(I,4)
      DO140J=1,3
      FORCE(I,J)=FORCE(I,J)+FINT(I,J)
      STRAN(I,J)=STRAN(I,J)+SINT(I,J)
 140  CONTINUE
C
C                                         PRINT DISPLACEMENT OF NODES
C
      DO120I=1,NP
      CORD(I,1)=CORD(I,1)+DIS(1,I)
      CORD(I,2)=CORD(I,2)+DIS(2,I)
 120  CONTINUE
      RETURN
      END
      SUBROUTINE SIGBAR(ANETA,MM,NI,NII)
      COMMON/CONTR/TITLE(12),NP,NE,NDF,NCN,NLD,NMAT,NSZF,NPRE,LMCN
      COMMON/CONTS/CORD(300,2),NOP(472,3),ORT(1,2)
      COMMON/CONTU/DELX,RAD,H1,H2,AD,Y
      COMMON/ABF/FORCE(410,4),STRAN(410,3),SIGB(410),INP(410),FFST(410)
      COMMON/CDE/DIS(2,300)
      COMMON/BCD/MB,NNN
       DO10N=1,NE
      A=(FORCE(N,1)-FORCE(N,2))**2
      B=(FORCE(N,2)-FORCE(N,4))**2
      C=(FORCE(N,4)-FORCE(N,1))**2
  10  SIGB(N)=SQRT((A+B+C)/2.0+3.0*FORCE(N,3)**2)
  11  CONTINUE
      SIGBM=0.0
      I=0
      DO20N=1,NE
      IF(INP(N).EQ.1)GOTO20
      IF(SIGB(N).LE.SIGBM)GOTO20
      SIGBM=SIGB(N)
      I=N
  20  CONTINUE
      IF(MM.EQ.0)GOTO40
      IF(SIGBM.GE.0.995*Y)GOTO30
      GOTO60
  30  INP(I)=1
      LMCN=1
      PRINT70,I
      GOTO11
  40  CONTINUE
      IF(SIGBM.GE.0.995*Y)GOTO50
      ANETA=Y/SIGBM
      GOTO60
  50  INP(I)=1
```

```
      LMCN=1
      MM=1
      PRINT70,I
      GOTO11
   60 CONTINUE
   70 FORMAT(I5,22H TH ELEMENR HAS YIELDED)
      IF(NI.EQ.NPR)GOTO500
      IF(NI.EQ.NII)GOTO500
      GOTO700
  500 CONTINUE
      NII=NII+NNN
      PRINT101
C
C                                 WRITE ALL STRESS COMPONENTS
C
      DO600N=1,NE
      PRINT111,N,(FORCE(N,I),I=1,4),SIGB(N),(STRAN(N,I),I=1,3),EFST(N)
  600 CONTINUE
  101 FORMAT(1X,*ELEMENT*,3X,*X-STRESS*,6X,*Y-STRESS*,5X,*XY-STRESS*,6X,
     1*Z-STRESS EFFECT.STRESS*,6X,*X-STRAIN*,6X,*Y-STRAIN*,5X,*XY-STRAIN
     2 EFFECT.STRAIN*)
  111 FORMAT(I5,5F14.4,4F14.6)
      PRINT100
      DO120M=1,NP,2
      ML=M+1
      PRINT110,M,(DIS(J,M),CORD(M,J),J=1,2),ML,(DIS(J,ML),CORD(ML,J),J=1
     1,2)
  120 CONTINUE
  100 FORMAT(///,15X,29HDISPLACEMENTS AND COORDINATES)
  110 FORMAT(2(I5,4F15.6))
      LMCN=0
  700 CONTINUE
      RETURN
      END
```

```
      DIS(2,J)=X(III+1)
   90 III=III+2
C
C                                          CALCULATE ELEMENT FORCES
C
      DO150I=1,NE
      FINT(I,4)=FORCE(I,4)
      DO150J=1,3
      FINT(I,J)=FORCE(I,J)
      SINT(I,J)=STRAN(I,J)
  150 CONTINUE
      DO200NC=1,NE
      CALL STBAM(NC)
      DO260I=1,NCN
      M=NOP(NC,I)
      IF(M.EQ.0)GOTO260
      K=(I-1)*NDF
      DO240J=1,NDF
      IJ=J+K
  240 R(IJ)=DIS(J,M)
  260 CONTINUE
      IA=K+NDF
      DO300I=1,3
      FORCE(NC,I)=0.0
      STRAN(NC,I)=0.0
      DO300J=1,IA
      FORCE(NC,I)=FORCE(NC,I)+BB(I,J)*R(J)
      STRAN(NC,I)=STRAN(NC,I)+AA(I,J)*R(J)
  300 CONTINUE
  200 CONTINUE
      DO330I=1,NE
      IF(INP(I).EQ.0)GOTO320
      COM=ORT(1,1)/(1.0+ORT(1,2))
      COM1=ORT(1,2)/(1.0-2.0*ORT(1,2))
      HP=0.0
      COMS=2.0*(1.0+2.0*(1.0+ORT(1,2))*HP/(ORT(1,1)*3.0))*SIGB(N)**2/3.0
      DJ=FINT(I,1)+FINT(I,2)+FINT(I,4)
      DSIGX=FINT(I,1)-DJ/3.0
      DSIGY=FINT(I,2)-DJ/3.0
      DSIGZ=FINT(I,4)-DJ/3.0
      CAB=COM*(COM1-DSIGX*DSIGZ/COMS)
      CAC=COM*(COM1-DSIGY*DSIGZ/COMS)
      CAD=COM*(-DSIGZ*FINT(I,3)/COMS)
      FORCE(I,4)=CAB*STRAN(I,1)+CAC*STRAN(I,2)+CAD*STRAN(I,3)
      GOTO330
  320 FORCE(I,4)=ORT(1,2)*(FORCE(I,1)+FORCE(I,2))
  330 CONTINUE
      DO500I=1,NE
      ZYX=STRAN(I,1)**2+STRAN(I,2)**2+0.5*STRAN(I,3)**2
      YZX=SQRT(2.0*ZYX/3.0)
```


Single Pass Cold Rolling Mill
Initial Thickness = 0.013"
PPER = 800
PPER
PPER = 900
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0
10%
20%
30%

APPENDIX-D

```
C       NP=NO. OF NODAL POINTS
C       NE=NO. OF ELEMENTS
C       NB=NO. OF RESTRAINED BOUNDRY NODES
C       NDF=NO. OF DEGREES OF FREEDOM PER NODE
C       NCN=MAX. NO. OF NODES PER ELEMENT
C       NLD=NO. OF LOAD CASES
C       NMAT=NO. OF MATERIAL TYPES
C       NSZF=NO. OF EQATIONS IN THE SYSTEM
C       L1=LOAD CASE COUNTER
C       COORD=NODAL POINT COORDINATE ARRAY
C       NOP=ELEMENT CONNECTION ARRAY
C       IMAT=ELEMENT MATERIAL TYPE ARRAY
C       ORT=ELEMENT TYPE MATERIAL PROPERTIES
C       NBC=RESTRAINED BOUNDRY MODE NUMBERS
C       NFIX=BOUNDRY CONDITION TUPE
C       R1=LOAD VECTOR
C       SK=RECATANGULAR MATRIX FOR EQUATIONS
        COMMON/CONTR/TITLE(12),NP,NE,NDF,NCN,NLD,NMAT,NSZF,NPRE,LMCN
        COMMON/CONTS/CORD(300,2),NOP(472,3),ORT(1,2)
        COMMON/CONTT/IMP(600),X(600),B(600)
        COMMON/CONTU/DELX,RAD,H1,H2,AD,Y
        COMMON/ABF/FORCE(410,4),STRAN(410,3),SIGB(410),INP(410),TFST(410)
        COMMON/ABG/SK(310,14)
        COMMON/BCD/MB,NNN
        COMMON/XYZ/BB(600)
        MB=14
        NCN=3
        LMCN=0
        NII=1
        READ1,NNN
        READ1,NPROB
C
C                                     LOOP ON NO. OF PROBLEMS
C
        DO400NPR=1,NPROB
C
C                                     READ INPUT GEOMETRY AND PROP. AND B.C.&S
C
        CALL GDATA
   400  CONTINUE
        CALL FLUN(32000)
        DO5I=1,NE
        TFST(I)=0.0
        INP(I)=0
        DO5J=1,3
        FORCE(I,J)=0.0
        STRAN(I,J)=0.0
    5   CONTINUE
        N1=1
```